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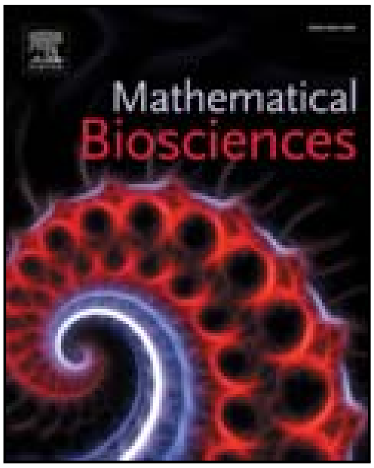

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Highlights

- Dynamical system modeling
- Dynamic behavior of the green, floral and woody biomass
- Mediterranean ecosystem
- Mathematical Formulation of Ecological Systems

<PE-AT>

BEHAVIOR OF PYROPHITE SHRUBS IN MEDITERRANEAN TERRESTRIAL ECOSYSTEMS (I): POPULATION AND REPRODUCTIVE MODEL

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Abstracts
The mathematical submodel ULEX is used to study the dynamic behavior of the green, floral and woody biomass of the main pyrophite shrub species, the gorse (*Ulex parviflorus Pourret*), and its relationship with other shrub species, typical of a Mediterranean ecosystem. The focus are the ecological conditions of post-fire stage growth, and its efficacy as a protective cover against erosion processes in the short, medium and long term, both in normal conditions and at the limits of desertification conditions. The model sets a target to observe the behaviour and to anticipate and consequently intervene with adequate protection, restoration and management measures.

Keywords: biomass, desertification, flow equations, flowers, fruits, population, pyrophite plants, state equations,

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1. INTRODUCTION

The garrigue (*carrascal*) is defined as forest formations in which the oak or *Quercus rotundifolia* dominates acting as a key element of the tree layer. The oak is the most representative tree in western Mediterranean vegetation, especially when the ombroclimate is dry. Garrigue degradation leads to the emergence of the lentiscal (*Querco-Lentiscetum*) as a first stage of substitution, or scrubs such as (*Rosmarino-Ericion*). In this case rosemary (*Rosmarinus officinalis*), heather (*Erica multiflora*), and monk's crown (*Globularia alypum*), gorse (*Ulex parviflorus*), and albada (*Anthyllis cistoides*) emerge giving spectacular flowering in the mountains for much of the year. These formations are highly variable. Currently, and as a result of degradation and intense reforestation, much of the garrigue is occupied by pine forests of Aleppo pine (*Pinus halepensis*). This pine grows naturally in the area as part of the serial stages of garrigue development, both as a mastic tree as scrub. However, when it occurs naturally without human intervention, it is not as dense in populations such as those present in the territory of Mediterranean mountain ecosystems near the coast and these naturally occurring pine forest reforestations should not be regarded as forest,. Obviously, reforestation in areas without vegetation and ruined soils is positive but counterproductive if presented to repopulate areas where mastic trees and shrubs are well structured.

Fire is an important ecological factor in all Mediterranean regions (Naveh, 1974, 1975, 1984; Naveh and Kutiel, 1989). The effects of fire on ecosystems are complex because the plant response is very variable depending on the type and time of the fire, the capacity for post-fire regeneration (from the re-sprouting required to facilitate seeding), as well as the status of pre-fire vegetation. The recurrence of the fire, as it may interfere with the life cycle of the species underscoring the risks of degradation of ecosystems, has emerged as a key factor in shaping the landscape in Mediterranean conditions. However, other factors related to land use affect both the structure and the floral composition of the various formations and therefore the dynamics of post-fire vegetation (Trabaud, 1981).

Pyrophyte plants are those species that are fire resistant and are favoured by fire. Pine is a heliophilous pyrophite that coexists well with other pyrophites such as gorse, contributing greatly to the spread of fires. The forest fuel can be light, consisting of leaves, grass and small bushes, and is characterized by burning and being consumed quickly, or it is heavy, consisting of trunks, branches, roots, and is consumed slowly. Obviously, the better fuel will be drier, because green foliage decreases the velocity of propagation, and combustibility depends on availability and continuity, of both horizontal and vertical flammable plant matter. Conifers, due to internal resin content and their fruits, the cones that can jump great distances when they burn, spread the fire much better than hardwoods. The combustion process of wood that propagates near the fire by convection is lifted by the air from the ground up, as warm air is lighter than cold, and also by columns of quickly rising smoke. Heat radiation is also propagated through the air molecules around the fire, or by conduction through the fuel molecules of plants, or by the soil itself. In recent years, fires have acquired great virulence mainly because the number of visitors has increased in mountain ecosystems. It is significant that the number of fires increases significantly at weekends, holidays and vacation times. Moreover, the firewood and scrub is not used practically as fuel, so the pyrophite material accumulates on the mountains. The accumulation of solid waste in uncontrolled landfills and or dispersed in the bush, especially glass, provide foci for starting fires. To this is added the Mediterranean climate, characterized by long hot summer droughts, compounded by climate change, each of which facilitates the desertification of these areas. When an extreme hot and dry west wind blows, ideal conditions are created for the emergence of numerous outbreaks of fire. In such weather, the atmospheric temperature varies depending on the altitude, so that for every 100 meters the temperature decreases by about one degree. Coastal breezes can change the temperature by several degrees from one nearby point to another. In the days when weather conditions are appropriate, the fire risk can be several times higher than normal. The study of this species is important to understand its relationship with fire. This species has persistent seed banks in the soil and its germination it can be stimulated by fire. These seeds remain viable in the soil after the fire and in some cases in high densities. At present, management techniques are used involving burning and slashing of mature populations of this species. One needs to take into account on the one hand the alteration produced in the environment using these techniques, as well as the effects on the soil, and secondly, how this species regenerates. The slash and burn treatments used to control gorse, are considered very efficient, but can lead to gorse not regenerating in the treated areas and being displaced by other species, so endangering the survival of the species.

The response of ecosystems to forest fires involves special attention to the biology of the gorse (*Ulex parviflorus*) as the ultimate goal of developing a scientific basis for the use and/or control of this species, including its restoration and the prevention of fire.

2. *ULEX PARVIFLORUS POURRET*, A PYROPHITE MEDITERRANEAN PLANT

Gorse (*Ulex parviflorus Pourret*) is a Fabaceae (family *Fabaceae*). There are about 20 species native to the European Atlantic territory and the western part of the Mediterranean region. The *Ulex parviflorus Pourret* is a pyrophite species, and is the first colonizer in post-fire stages of ecological regeneration (Figure 1).

Figure 1: The *Ulex parviflorus P.*

The study of their behavior can provide important conclusions that can have a significant impact on sound management of the territory, both before the fire (prevention) and after it (regeneration). These plants are very important for:
1. Holding the soil and preventing erosion.
2. They are excellent nitrogen fixers. They are abundant in unfavourable environments because their roots have small nodules, corresponding to symbiotic bacterial colonies fixing atmospheric nitrogen. This explains the ability of legumes in poor environments, which makes these plants environmentally interesting as they enrich the ground with nitrogen compounds that are always in short supply. They improve the land they occupy, and supply plant products, fodder and seeds, and wealth in proteins.
3. They provide seed banks at very shallow depths**.** This accumulation of seeds has a major influence on the dynamics of plant communities and more specifically in the regeneration of plant cover.
4. The fundamental role of the species *Ulex parviflorus Pourret* is manifested in its close relationship with forest fires, and together with the *Pinus halepensis* is the most important pyrophite species of the Mediterranean forest (Baeza, Raventós, Escarré and, Vallejo, 2006; De Luis, Baeza, Raventós, González-Hidalgo, 2004; Dimitrakopoulos and Panov, 2001; Papió and Trabaud , 1990, 1991). This pyrophite character is evident in:
1) Increased level of flammable material.
2) Germination of seeds that are favoured by the fires and the lack of competition in relation to other species which are removed by fire.
Therefore, it is necessary to know its development to find out at what stage their life cycle can be started to establish their control in their environment.

2.1. Characteristics

1) **Ecology and distribution:** Its distribution area is the Western Mediterranean. (*Rosmarinetalia, Lavanduletalia stoechadis*). In Catalonia it is in the maritime regions, in certain areas and the altitude varies between 0-1000 m. In the Valencian Community: From the port of Morella and Baix Maestrat, to the mountains and the *Dianic "Serrànic and Lucèntic Territory"* the altitude varies between 0-1250 m. In the Balearic Islands: Ibiza for example, the altitude is between 0-300 m. It is a very common and abundant bush in the Valencian Community, provided there is a maritime influence on the environment (*maresía*) as it is very sensitive to the more extreme continental climate. While it is very common in areas near the coast, it gradually dies out as it progresses into the country and stops feeling the influence of wind from the sea. It is indifferent to the nature of the substrate soil, but vegetates best in soils rich in bases and, like most of perennials of the family prefers deep soils. Undoubtedly, in the western Mediterranean area, the common gorse (*Ulex parviflorus Pourret*) is the main pyrophite and therefore the study of its behavior following fire can provide conclusions useful in the proper management of the territory, both before the fire (prevention) and after it (regeneration). If we can understand through mathematical modelling the plant's biological strategies we can quantify the plant's development over time and space, and we will be closer to achieving an orderly influence on the complexity of the terrestrial Mediterranean ecosystem.
2) **Habitat:** The *Ulex parviflorus Pourret* is a Nanophanerophyte plant that develops in dry scrub areas especially in the Western Mediterranean. It lives at an altitude of 0-1250 meters.
3) **Soil types:** It is a heliophilous plant that lives in open sunny places and it can be found in large deforested areas in the Mediterranean mountains. Because of winter flowering, this plant is not found inland where the winters are too cold. It lives in coastal regions, in poor, dry and stony soils. The gorse is a tall bush, mainly consisting of seeders, occupying ground that may be more or less disturbed, in which the species *Ulex parviflorus* is clearly dominant over the other species. In the early stages of its presence in an area, which we describe as youth communities, it can contribute 45% of the total coverage. Although the level of climatic protection that facilitates these phases is suitable,the soil layer is not too deep to prevent the presence of invasive opportunistic species such as the one discussed here,
4) **Flower characteristic:** The flowers have the following characteristics: a) Solitary or fasciculate on thorns, yellow flowers. b) Size 6 to 8 mm. c) Zygomorphic flowers, pentamerous with 5 welded sepals and five free and unequal petals. d) The typical papilionacea flower is made up of an upper outer petal generally larger than the other banner petal; on the flanks there are a pair of side petals covered by the banner petal called wings, coated in turn to another couple, the two forward petals that touch at their edges and form the keel. e) In *Ulex*, the banner and keel are similar in length to the calyx, and shorter than the wings. f) Chalice 6-8 (10.5) mm with overlapping hairs or glabrescent. g) Bracteoles 1-1.5 mm wide. h) Thorns 1- 2 (3) cm, straight or curved, gray-green and disseminated. i) Androceo consists of 10 stamens (9 soldiers + 1 free). j) Gyneceum formed by a single carpel, a superior ovary surmounted by a style and stigma within a stamen tube (formed by welded stamens). k) The number of ovules varies from 2 to more numerous and inserts alternating in two rows on a single placenta. It flowers from the months of December to February forming large associations (winter flowering).

5) **Characteristics of fruit and seed:** The fruits and seeds have the following characteristics: a) The fruit is a legume, a dry dehiscent fruit, with a sheath that opens in two valves. Size: 7 - 12 x 3-6 mm. b) The seed contains a large embryo, with or without endosperm. c) The seeds are ejected violently (with an explosion) to open the leaflets of the legume. This phenomenon is because the shells are dried and tend to be wound helically about themselves along the lines of dehiscence. This opening phenomenon occurs in moments of heat pulses and spreads the seeds around. The fruiting period takes place during the months of December to February (May at the latest).
6) **Leaf characteristics:** It is a glabrescent shrub, with leaves reduced to small scales at the base of the spines (less than 1 mm diameter green ash). Simple and small leaves.
7) **Most common method of pollination:** The most common method of pollination is insect pollination. The study of pollination of *Ulex parviflorus* corroborates other studies which also states that the optimal pollination period occurs during the day between 08:00 and 18:00. So, the time available for pollination of any flower can be as much as 10 hours a day, for up to eight days, but is more effective for 4 days (Richards, 1986). The flowering of this species is early, and there is a very large decrease in competitiveness with other species and consequent effectiveness in seed formation when pollinated by bees. *Ulex* experiences these difficulties because it competes in the pollination with albaida (*Anthyllis cytisoides L.*), with rosemary (*Rosmarinus officinalis L.*) and thyme (*Thymus vugaris L.*).
8) **Method seed dispersion:** By studying the dispersion of the seeds of *Ulex parviflorus*, it is found that the structure of the fruit is intimately related to the way it spreads, that is, making an explosive dehiscence (autochory). The different layers of the pericarp of the fruit, contract to varying degrees as they dry, so considerable tensions between them (hygroscopic movements) are established. When these stresses exceed the cohesion of the cell walls, they separate very suddenly. The leaflets of legumes are wound on themselves helically so ejecting seeds around; the separation is normally violent and fast. The seeds of *Ulex parviflorus* are violently expelled from their fruits (explosive dehiscence) then falling to the ground by gravity. If this was the only form of dispersion, all seeds should be located near the parent plant, but this is not the case, raising the possibility of secondary dispersion (Beattie & Culver, 1981). The ants are responsible for this secondary dispersion. This double dispersion maintains the population, so establishing a double seed bank. Seeds that have been transported to anthills and manipulated by ants germinate better, indicating an effect of aryl, that is, there is a greater preference for aryl seeds. The seeds of *Ulex*, benefit then from two processes of dispersion, one primary and another secondary, by the ants. The prominence of the ant and their density in the local ecology may condition the appearance of new plants of *Ulex*.
9) **Phytosociological associations:** Gorse lives in thermomediterranean and mesomediterranean floors. Gorse appears in degraded areas of the Garrigue, which leads to the appearance of kermes oak with mastic (*Querco - Lentiscetum*) and rosemary (*Rosmarino - Ericion*) in which the rosemary (*Rosmarinus officinalis*), heather (*Erica multiflora*), the crown friar (*Globularia alypum*), gorse (*Ulex parviflorus*) and albada (*Anthyllis cytisoides*) are the most common species. Garrigue uniformity is broken up by the topography or weather conditions. In the middle mesomediterranean floor level, in areas more tempered

by some influence from the humid winds from the Mediterranean, gorse (*Ulex parviflorus*) is common and with a nuance towards *Rubio - Quercetum rotundifoliae*. It is possible also to find gorse in the Valencian scrub lands, showing great consistency and appearing with heather, rosemary and albada.

2.2. Importance of this species as pyrophite

Ulex parviflorus is considered a very flammable species during the summer, since it conserves its dry elements in branches, stems, etc. The caloric power (energy that can be released in a combustion) of *Ulex parviflorus* would be found in a mid-point along with *Cistus*, *Lavandula*, *Quercus*, *Stipa*, *Thymus*, etc. It is a species that presents a great amount of necromass accumulated at the plant's foot. In advanced stages of maturity, the proportion of necromass may exceed 50% of the total biomass. The distribution of the necromass in height also poses many problems, since in these conditions the height reached by the flames in the fire is much greater. It is a forced germinating species, which is never regenerated by regrowth. Only under mild pruning conditions can the biomass levels be restored. After the passage of fire a high germination rate can be observed that at three months after the fire can be of 60 seedlings per m^2, decreasing to an average of 7.3 individuals per m^2 17 years later. A mass of the gorse land is easily installed in the area occupied by other crops abandoned after the fire. In these conditions, the other species have been removed for the installation of the next crop. After the passage of the fire, *Ulex parviflorus* does not find any type of competition and displaces the rest of germinators, except for the arboreal ones like in the case of pines. In mature pine forests, with higher levels of shade, the growth of *Ulex parviflorus* are very poor and with lower seed yields, however after the fire the seeds that germinate produce vigorous individuals that will form a high density of gorse plants. The seed banks studied show that there are no differences in the seed banks of two formations with different stages of development: the number of seeds per square meter is 319 s m^2 in the 3-year-old brood and 324 s/m^2 in 9-year-old plants. The fact of having an equal number of seeds in juvenile formations, which had not yet flowered, compared to mature formations in which there have been several blooms, indicates that *Ulex parviflorus* presents a permanent bank of seeds in the soil, which is not exhausted at the first germination after the disturbance. In the case of *Ulex europaeus L*. the seeds may remain latent and viable in the soil for a period of at least 30 years due to the presence of a hard waterproof cover. Preliminary evidence indicates that seed production in the 5 to 7-year-old formations is much higher than in the more mature ones. This greater production will contribute to increase the seed banks, stabilizing when the production reaches the stage of maturity. Therefore, control treatments of the gorse in intermediate ages, before the maximum seed production, could contribute to reduce the seed bank and the regenerative capacity of the species. The low percentage of changes observed in plant remains dropped from *Ulex parviflorus* together with the low input of fresh organic matter suggests that the decomposition and humidification of the soil material on the soil surface is not the main process involved in the recycling of nutrients, which are stored in the biomass of the shrub. In studies carried out in Mediterranean shrublands, frequently subjected to fire, this is proven to be an important factor in nutrient restitution, since the rapid mineralization of large amounts of organic matter as a result of a fire increases the productivity of ecosystems up to a certain level of mineral elements.

3. THE STUDY AREA

The plots studied are located within the province of Castellón (Comunidad Valenciana, Spain). The experimentation plot is in the *Desert de les Palmes* (*Monte Bartolo*) (Figure 2).

Figure 2: Experimentation site.

The validation plot is in *Benlloch* (*Serra d'En Garceràn*). The chosen areas have similar lithology, soils, topography, climatology ... , and are located in different zones that have been considered representative of the Mediterranean terrestrial ecosystem. Validation is checked in regard to space and time. Each plot is 100 m^2 and selected so that the number of plants around *Ulex parviflorus* was the smallest possible in order to achieve better sampling and reduce interference. All specimens present inside the plot were placed on a plane (Table 1).

TABLE 1
Characteristics of plots

	Plot of experimentation	**Plot of validation**
Location	Desert de les Palmes	Serra d' En Garceràn
UTM	31 T BE 4741	30 T YK 5458
Altitude (m)	550	500
Reforestation	Yes	No
Climatic characteristics[1] (*)	Average annual temperature:15,2 ºC Average temperature of the coldest month (January): 3º C Average temperature of the warmest month (July): 29,4 ºC Average annual precipitation:540 mm	Average annual temperature:14,8 ºC Average temperature of the coldest month (January): 1,7 ºC Average temperature of the warmest month (July):30,8 ºC Average annual precipitation:619 mm

[1] Since September 1991 an Automatic Meteorological Station is in operation on top of Monte Bartolo (735 meters above sea level and 6 km from the sea) forming part of the network of observatories of the Universitat Jaume I. Its function is: to record of all climatic and environmental variables that it is equipped to measure automatically (including solar radiation, infrared radiation, soil and air temperature, atmospheric pressure, winds in direction and force, relative humidity, rainfall, etc.) These are the main exogenous variables taken into account in the model. All registers correspond to analogue inputs in 16 channels, with continuous scanning every two seconds, which can be processed, in any interval chosen from 5 minutes, using averages, maxima and minima, as well as filters of alternating values. It also has a digital input line for events such as rain and relative humidity. Data is registered on a hard drive using a duplex RS232C and data can be downloaded in ASCII format.

List of plants	*Ulex parviflorus* Pourret *Cistus monspeliensis* L. *Cistus albidus* L. *Pistacia lentiscus* L. *Quercus coccifera* L. *Juniperus oxycedrus* L. *subsp. oxycedrus* *Pinus halepensis* Miller *Rosmarinus officinalis* L. *Chamaerops humilis* L. *Inula viscosa* L. (Aiton)	*Ulex parviflorus* Pourret *Cistus albidus* L. *Cistus clusii* Dunal *Erica multiflora* L. *Quercus coccifera* L. *Thymus vulgaris* L. *Rosmarinus officinalis* L. *Santolina chamaecyparissus* L. *Ononis natrix* L. *subsp. natrix* *Lavandula latifolia* Medicus *Ruta angustifolia* Pers. *Helichrysum stoechas* (L.) Moench

(*)The climatic characteristics, have been obtained on the basis of 33 years of observations (1941-1974). Source of data: Quereda (1976, 1985)-

4. THE ULEX MODEL

The ULEX model aims to determine the development (germination, fruiting, etc.) of this pyrophite, and tries to extrapolate the study to other plants with similar characteristics. To do this it is necessary to connect the developmental model showing which species are representative at which time of the year and play an important role in the genesis of fires and post-fire ecosystem evolution.
The MARIOLA calculation system is the basis for calculating the specific model ULEX. The MARIOLA model (Mateu, Usó and Montes, 1998; Usó-Domènech, Villacampa, Stübing, Karjalainen and Ramo, 1995; Usó-Domènech, Mateu and Lopez, 2000; Usó-Doménech, Nescolarde-Selva, Lloret-Climent and Meng, 2016.), so called for having taken as the base the mountainous terrestrial ecosystem of the Sierra de Mariola (Alicante, Spain), is a dynamic model based on System Dynamics (Forrester, 1961), in which differential equations (state equations) are set up hypothetically, and flow variables are obtained from field observations using multiple regression equations (Cortés, Villacampa, Mateu and Usó-Doménech, 2000; Nescolarde-Selva, Usó-Doménech, Lloret- Climent,. and González-Franco, 2015; Usó-Domènech, Mateu and Lopez, 1997, 2000; Usó-Doménech, Nescolarde-Selva, Lloret-Climent and Meng, 2016).
The development of the ULEX model can predict the effect that the climatic changes of several variables produce on others. This model requires the following conditions:
1) A causal structure based on climatological, ecological and edaphological theories.
2) Functions determining the behavior of the variables under study.
3) A validation procedure, which compares results obtained by the model and those obtained by experimentation.
The main characteristics of ULEX model (Jorgensen, 1988) are.
1) It is deductive.
2) It is compartmental.
3) It is stochastic-deterministic.
4) It is holistic.
5) It is dynamic.
6) It is causal.
7) It is biodemographic and biogeochemical.

8) It is a non-linear model.
9) The model disaggregation is high in order to be able to study behaviors in morphological and ecophysiological levels.
The ULEX model presents two options of calculation:

a) The **BULEX** model is based on the study of the behavior of plant biomass. Its main subprograms are the following:
1) The **BIOSHRUB** program, which builds the formulas for the Mediterranean bush biomass. It stores the Mediterranean shrub allometric formulas (Usó, Mateu, Karjalainen and Salvador, 1997).
2) The **SELEGO** program, which builds the flow equations. The growth equations of the total, green and woody biomass can be found as a particular case of flow equations, which forces the construction of a subroutine of growth, which is calculated from the simple difference between the existing biomass in period i and in period i+1.
3) The **VEGETATION** program, which is the numeric resolution of the flow equations and the resolution of the system of first order ordinary differential equations using Euler's method. An option is to integrate the Runge-Kutta method if necessary for more precision, most of all when the model disaggregates a level using ecophysiological variables. The starting variables are obtained by a Montecarlo simulation.

The flow chart of the BULEX model can be seen in figure 3.

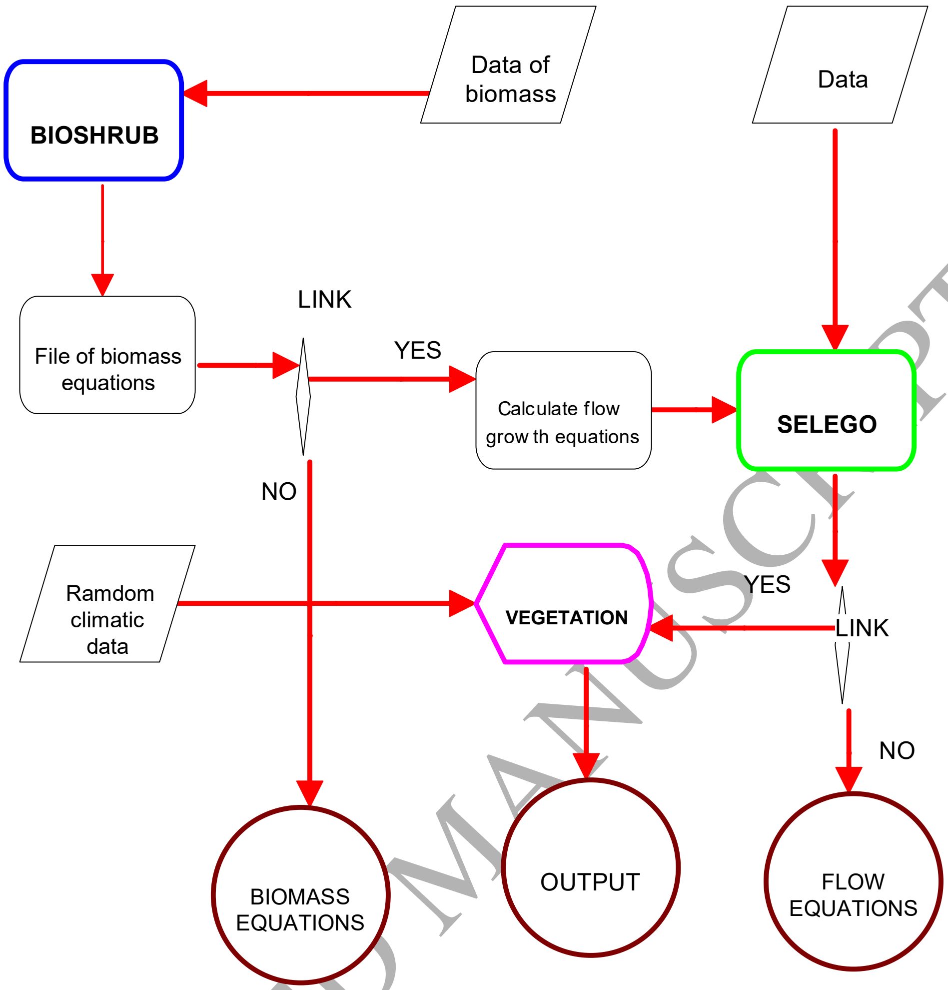

Figure 3. Flow chart of BULEX model.

b) **PULEX**, population model that studies and analyzes the behavior of the plant population or the population of its constituent parts (leaves, flowers, fruits, seeds, etc.). Its main subprograms are the following:

1) **SELEGO** program.
2) **VEGETATION** program

The flow chart of the PULEX model can be seen in figure 4.

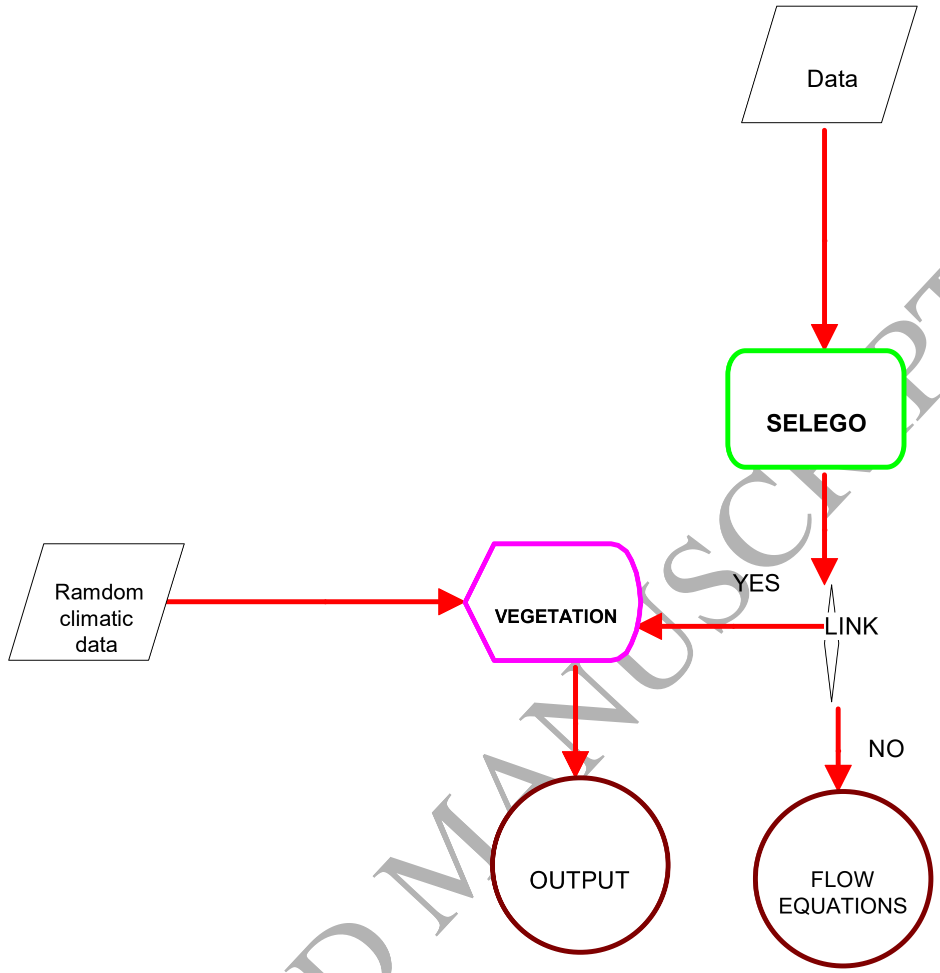

Figure 4. Flow chart of PULEX model

There are three sub-models, of the model ULEX:

1) The submodel for the reproductive biomass or BULEX.
2) Reproductive population submodel or RPULEX.
3) Plant population submodel or PULEX

4.1. Submodel for reproductive biomass BULEX

Using the calculation system BULEX. The state variables are (Table 2):

Table 2
State variables

Variable symbol	*explanation*	*unit*
BT	Total biomass	g
BFL	Floral biomass	g
BFRUT	Fruit biomass	g
BLIT	Biomass fallen on the ground	g

Flow variables are as follows (Table 3):

Table 3
Flow variables

Variable symbol	*explanation*	*unit*
CRBT	Growth rate of total biomass	g/time
DBT	Destruction rate of total biomass	g/time
CRFL	Growth rate of floral biomass	g/time
DFL	Rate of destruction of floral biomass	g/time
CRFRUT	Growth rate of the fruit biomass	g/time
DFRUT	Rate of destruction of fruit biomass	g/time
CRLIT	Growth rate of soil biomass	g/time
DLIT	Decomposition rate of soil biomass	g/time

The exogenous, auxiliary variables and parameters are (Table 4):

Table 4
Exogenous and auxiliary variables and parameters

Variable symbol	*explanation*	*unit*
PLU	Precipitation	l
T	Environmental temperature	ºC
H	Environmental humidity	%
RS	Solar radiation	kW/m^2
DIVI	Wind direction	Sexagesimal degree

4.2. Reproductive population submodel RPULEX
Using the calculation system PULEX. The state variables are (Table 5):

Table 5
State variables

Variable symbol	*explanation*
PGEM	Population of flower buds
PFLOR	Population of flowers
PFRUT	Population of fruits

The flow variables are as follows (Table 6):

Table 6
Flow variables

Variable symbol	*explanation*
NGEM	Birth of flower buds
DGEM	Destruction of flower buds
NFLOR	Birth of flowers
DFLOR	Destruction of flowers
NFRUT	Birth of fruits
DFRUT	Destruction of fruits

The exogenous, auxiliary variables and parameters are (Table 7):

Table 7

Exogenous and auxiliary variables and parameters

Variable symbol	*explanation*	*unit*
PLU	Precipitation	l
T	Environmental temperature	ºC
H	Environmental humidity	%
VEVI	Maximum wind speed	km/h max
DIVI	Wind direction	degrees
RS	Solar radiation	kW/m^2

3.3. Plant population submodel PULEX

Using the calculation system PULEX. The state variables are (Table 8):

Table 8
State variables

Variable symbol	*explanation*
PSEM	Population of seeds on the ground
PGERM	Population of germinated seeds
POB	Plant population

The flow variables are as follows (Table 9):

Table 9
Flow variables

Variable symbol	*explanation*
DISP	Seed dispersion
GERM	Seed germination
AGERM	Non-seed germination
NAC	Birth of plants
ABORT	Germinated seeds aborted
MORT	Death of plants

The exogenous, auxiliary variables and parameters are (Table 10):

Table 10
Exogenous and auxiliary variables and parameters

Variable symbol	*explanation*	*unit*
PLU	Precipitation	l
T	Environmental temperature	ºC
H	Environmental humidity	%
VEVI	Maximum wind speed	km/h max
DIVI	Wind direction	
NSEM	Average number of seeds per fruit	
TGERM	Germination rate	
TAGERM	Rate of non-germination	

5. EXPERIMENTAL METHODOLOGY

5.1. Measurement of biomass

5.1.1. Theoretical fundament

The dynamics of a system as it has been proposed can be modeled by means of a system of differential equations. If we call BG the biomass growth over the period of time $\left(t_{i+1} - t_i\right)$, then:

$$BG = B\left(t_{i+1}\right) - B\left(t_i\right), i = 1,2,...,n \quad (1)$$

For this reason it is necessary to know the biomass B in each period of time. The rate of increase in biomass is proportional to the apparent volume and is defined by the following dynamic differential equation:

$$\frac{dB}{dV_{ap}} = b\left(V_{ap} - c\right) \quad (2)$$

The global biomass (B) as a function of the apparent volume $\left(V_{ap}\right)$ is the result of the integration of the previous differential equation:

$$B = c + ae^{bV_{ap}} \quad (3)$$

This equation is exponential and therefore nonlinear in the variable, but linear in the parameter. It can be linearized as follows: by defining $V'_{ap} = e^{bV_{ap}}$, then the equation $B = c + V'_{ap}$ is a linear regression equation. In this equation, "a + c" represents the initial biomass, ie the biomass corresponding to an apparent volume equal to zero. It is clear that "a + c" should be zero in all cases, but experimental works generally show that it is not true due to experimental errors and deviations in function setting. It is also important to note that the sign of "a" must be the same as the sign of "b", otherwise $\frac{dB}{dV_{ap}}$ would be negative, and that does not make sense. Making the derivative of the previous equation we will have

$$\frac{\partial B}{\partial V_{ap}} = abe^{\left(bV_{ap}\right)} \quad (4)$$

and

$$B\frac{\partial^2 B}{\partial V_{ap}^2} = ab^2 e^{\left(bV_{ap}\right)} \quad (5)$$

Consequently, *the radius of curvature* (R) will be calculated as follows:

$$R = \frac{\left[1 + a^2 b^2 e^{\left(2bV_{ap}\right)}\right]^{\frac{3}{2}}}{a^2 be^{\left(bV_{ap}\right)}} > ab^2 e^{\left(2bV_{ap}\right)} \quad (6)$$

From the point of view of differential geometry, the radius of curvature of a point P in a curve is the inverse of its curvature K. The curvature K at a point P is the variation of the tangent angle in P on the arc of unit length. K is positive if P is in a concave and negative curve if P is on a convex curve. This allows to conclude that for positive values of "a" and "b" the defined curve will be concave, and the highest value of positive values corresponds to the highest value of the radii of curvature that is given for a given apparent volume and consequently the closest to a straight line. On the other hand, if "a" and "b" are negative, the curve will be convex, and these values correspond to a straight line. The biological interpretation of this fact is that for positive values of "a", the lower the apparent volume and the higher the *apparent density* $\left(\frac{B}{V_{ap}}\right)$. For negative values of "a" and "b", the apparent volume will be higher and the bulk density will be lower. Therefore, "a" and "b" have an apparent biological interpretation, but "b" has a larger influence on the equation since it is in exponential form. Therefore, the parameter "b" can be considered as an index showing the development of apparent density $\left(\frac{B}{V_{ap}}\right)$ with respect to apparent volume. The parameter "c" represents the asymptotic values of an exponential curve and in the case of a concave curve can be interpreted biologically as the maximum biomass that a plant can accumulate. Deriving the biomass B with respect to the three dimensions of space (x, y, z), we will have:

$$\begin{cases} \frac{\partial B}{\partial x} = \frac{\partial B}{\partial V_{ap}} \frac{\partial V_{ap}}{\partial x} = b\left(B - c\right)\pi yz \\ \frac{\partial B}{\partial y} = \frac{\partial B}{\partial V_{ap}} \frac{\partial V_{ap}}{\partial y} = b\left(B - c\right)\pi xz \\ \frac{\partial B}{\partial z} = \frac{\partial B}{\partial V_{ap}} \frac{\partial V_{ap}}{\partial z} = b\left(B - c\right)\pi xy \end{cases} \quad (7)$$

The solution of this system of differential equations is the following:

$$B = b\left(B - c\right)xyz = b\left(B - c\right)V_{ap} \quad (8)$$

This equation leads to the equation of the apparent density of the plant

$$\rho_{ap} = \frac{B}{V_{ap}} = b(B - c) \quad (9)$$

The apparent volume is the volume of the circular cylinder defined by its height (h) and its diameter (d):

$$V_{ap} = \frac{\pi d^2 h}{4} \quad (10)$$

Ten taxa (Usó, JL, Mateu, J., Karjalainen, T. and Salvador, P. 1997) were studied for representative Mediterranean shrubs (Stübing et al., 1989), which play an important role in scrub of the western Mediterranean regions, especially during the first ten years after a forest fire. These species are: *Bupleurus frucicescens* L., *Ulex parviflorus Pourret, Helychrisum stoechas* (L.) Moench, *Rosmarinus offcinalis* L., *Lavandula latifolia Medicus, Sedum sediforme* (Jacq.) Pau, *Genista scorpius* (L.) DC in Lam. y DC., *Marrubium vulgaris* L., *Thymus vulgaris* L., *Cistus albidus* L. After taking the measurements, the plants were cut at ground level and dried at constant weight. The actual biomass was obtained by simply weighing the dried plants on a monoplate balance (0.1% accuracy). 30 examples of each taxon were used, as well as 30 others for the validation process. We used the SELEGO software that provides the "best" regression equation defined based on the highest correlation coefficient of biomass and apparent volume. The results are summarized in the following table (Table 11)

TABLE 11
Regression parameter values

Taxon	c	c int. 95%	a	a int. 95%	b	r	s
Bupleurum fructicescens	-37150.3	[-40159.8, -34140.5]	37164.0	[34156.4, 40171.6]	0.01	0.98	6.81
Ulex parviflorus	**-825.5**	**[-974.2, -678.7]**	**846.4**	**[711.2, 981.5]**	**0.50**	**0.95**	**35.15**
Helychrisum stoechas	-401.7	[-447.7, -355.6]	422.7	[383.3, 462.1]	1.00	0.93	30.35
Rosmarinus officinalis	-367.0	[-418.1, -317.6]	381.8	[338.9, 424.7]	2.00	0.98	19.38
Lavandula latifolia	9.6	[0.9, 18.9]	19.6	[17.9, 22.0]	10.0	0.96	13.37
Sedum sediforme	144.2	[105.0, 183.5]	-141.9	[-181.7, -101.9]	-7.00	0.85	16.27
Genista scorpius	504.7	[434.8, 74.6]	-485.7	[-567.2, -404.3]	-1.00	0.95	25.10
Marrubium vulgaris	591.5	[475.5, 707.4]	-581.4	[-718.5, -444.4]	-1.00	0.93	3.47
Thymus vulgaris	46249.4	[34218.4, 58268.4]	-46247.3	[-58282.3, -34212.5]	-0.05	0.89	37.31
Cistus albidus	54752.7	[46703.9, 69901.6]	-54736.0	[-63797.2, -46674.8]	-0.01	0.98	36.05

r = correlation coefficient.
s = standard error of estimation.

A non-parametric statistical test was used to test the normality of the data with a confidence level of 95%. Table 12 shows the results of the validation.

TABLE 12
Results of validation

Taxon	d	d int. 95%	PM	s
Bupleurum fructicescens	5.10	[-5.09, 15.28]	38.9	3.67
Ulex parviflorus	**-4.58**	**[-17.30, 8.14]**	**63.75**	**4.58**
Helichyisum stoechas	1.92	[-11.88, 15.51]	128.75	4.93
Rosmarinus officinalis	0.08	[-16.26, 16.42]	111.42	5.89
Lavandula latifolia	-6.90	[-18.68, 4.88]	101.07	4.24
Sedum sediforme	0.96	[-0.62, 2.54]	5.24	0.57
Genista scorpius	-14.40	[-40.07, 11.27]	165.96	9.25
Marrubium vulgaris	-2.96	[-25.09, 20.12]	135.81	8.33
Thymus vulgaris	6.44	[-4.87, 17.75]	49.4	4.07
Cistus albidus	-1.46	[-14.83, 11.91]	100.51	4.83

d = mean difference between B (estimated biomass) and real biomass.
PM = average biomass B.

s = standard deviation of PM.
d int. 95% estimated interval d considering a confidence level of 95% with a Student t distribution value of 2.776.

The function $R = f(V_{ap})$ based on the radius of curvature equation has been solved. The radii of curvature of *Cistus* and *Thymus* are negative and correspond to convex curves. Positives are the radius of curvature of the *Ule*x and *Rosmarinus* and correspond to concave curves (Table 13).

TABLE 13
Radius of curvature for $V_{ap} = \{0,1\}$ in various species

Taxon	V_{ap}	R
Cistus albidus	0.00 1.00	-5.47363 -5.36524
Thymus vulgaris	0.00 1.00	-115.618 -104.616
Ulex parviflorus	**0.00** **1.00**	**200.543** **578.295**
Rosmarinus officinalis	0.00 1.00	1527.204 83382.33

The value of parameter *b* is close to zero for *Ulex parviflorus* (*b* = 0.5), *Thymus vulgaris* (*b* = -0.05) and *Cistus albidus* (*b* = -0.01) and very different *for Rosmarinus officinalis* (*b* = 2.0).
If *b* = 0 then *R* = ∞ and *K* = 0. It means that the corresponding curve equals the *X* axis (*apparent volume axis*) indicating that there might be a taxon that does not fit any environmental variation. The closer the parameter b to zero, the more resistant the taxon to environmental variations. Therefore, the axis of the Apparent Volume will be an *axis of total adaptation or maximum resistance*. If $b = 0$, the equation $B = ae^{bV_{ap}} + c$ becomes $B = a + c$. But $a \approx -c$, then $B = 0$ which means we have a taxon that has an apparent volume and no biomass, which is impossible and absurd. This means that there is no taxon completely resistant to environmental factors. This fact confirms the importance of parameter *b* as an *intrusivity index* that specifies the biological conditions of each plant through environmental conditions. If we observe the values of *b* in table 11, the extreme values of these parameters are 10 and -7, and in most cases they oscillate between 1 and -1. The value of *b* = 10 corresponds to *Lavandula latifolia Miller*, which has high proportions of lignified tissues. At the other extreme, the value of *b* = -7 corresponds to *Sedum sediforme*, a succulent plant with a high proportion of water accumulation. Therefore, it is possible to establish two strategies in the processes of accumulation of aerial biomass:

1) *Intrusive strategy* for high positive values of *b.*
2) *Extrusive strategy* for high negative values of *b.*

The two strategies correspond to the responses of plants to climatic factors.

5.1.2. Measurement of aerial biomass in *Ulex parviflorus*

In order to measure the biomass, we proceeded as follows, according to Usó et al (1995, 1997):

1. An experimental area of 100 m^2 has been selected in which 15 representative specimens of the species to be considered *Ulex parviflorus Pourret* have been studied at different stages of their development.
2. In order to determine the aerial biomass of *Ulex parviflorus*, measurements of maximum diameter and maximum height were taken in order to obtain the apparent volume of the plant, assimilated in a cylinder, by means of allometric equations previously established.
3. The counting of flowers and fruits has also been done visually, with a weekly frequency. Measurements have been repeated in the validation zone.
4. The height and maximum diameter of 15 specimens of *Ulex parviflorus Pourret*, at different stages of their development, have been measured with a 300 cm rigid tape with centimetric divisions.
5. These samples have been cut to ground level after having been previously labeled.
6. In the laboratory, these specimens have weighed in an electronic balance.
7. The samples were submitted to a desiccation process in an oven, at a constant temperature of 45º C, for 17 days.
8. The samples are weighed again.

5.2. Measurement of defoliation

1. The classical procedures are used with the modifications made by Usó et al (1995). The defoliation of *Ulex parviflorus* has been estimated in two areas: experimentation and validation. For this purpose, the previously selected samples have been used, to which the apparent volume has previously been measured.
2. To obtain data on defoliation, the flowers and/or fallen fruits were collected in a nylon mesh, with a 2 mm mesh, located under each specimen under study (Figure 5).

Figure 5. Measurement of defoliation

3. The samples collected weekly are subjected to a desiccation process in a hot air desiccator at constant temperature of 40 ° C for 48 hours and then weighed on a single platen scale.

5.3. Biomass decomposition

To study the decomposition of biomass, the methodology used by Ballini and Bonin (1995) is followed:

1. 120 samples of 2.5 grams each of flowers of the plant to be studied are placed in

bags of material not alterable in the range of time and conditions foreseen in the experience and that allow the passage to the decomposers. The bags are made of 15x15 cm nylon material with a 2mm weft.
2. The bags are left in the open air, on the ground or under the different taxon that are in the zone of experimentation, in order to be able to reproduce with the most possible fidelity, the natural conditions.
3. Three samples are collected weekly and weighed, subjected to the drying process explained above and weighed again.

The allometric equation obtained for Ulex parviflorus is as follows:

$$B = 846.4e^{0.50V_{ap}} + -825.5 \quad (11)$$

with a correlation coefficient of r = 0.95 and a standard deviation of s = 35.15.
The intrusivity coefficient b = 0.5 positive indicates a moderate intrusive strategy and being close to the value of zero, an approximation to the axis of maximum resistance or total adaptation. This is confirmed by the ecophysiology of the taxon and the fact of being a colonizer after forest fires.

5.4. Study of flowering
It is a winter flowering plant, it develops from mid-autumn to late winter. Up to 80 days with intense flowering has been observed in the same individual (Herrera, 1985). In spring after flowering, the main branches begin to grow forming new branches on the branches of the previous year.

1) Five specimens of *Ulex parviflorus* are selected at different stages of their development.
2) To obtain data on flowering have been studied 5 states that are as follows:
a) State 1: Flower bud.
b) State 2: Flower not open.
c) State 3: Open flower.
d) State 4: Corolla dries and begins the formation of the fruit.
e) State 5: Fruit formed.
3) Different flowering stages have been counted in branches oriented to the north, south, east and west; with a size of 20 cm.
4) The count has been weekly.

The flowering of this species is early and very numerous favoring the decrease of competitiveness with other species and the effectiveness in the formation of seeds to be pollinated by the bees. *Ulex parviflorus* manages to save its difficulties because it competes in periods of pollination with the *Anthyllis cytisoides L.*, *Rosmarinus officinalis L.* and *Thymus vugaris L.*

5.5. Study of fructification and seed dispersal
The study carried out on the fruiting period of this species from October to July reflects that from November, December, the fruits begin to form, reaching the highest number of fruits in the plant in March, April. From this period the expulsion of the seeds from the fruits begins. The primary dispersion period runs from late March to June, presenting a period of maximum dispersion in June. The growth of the plant coincides with the ripening of the fruits. After dispersion of the seeds, the plant enters a period of rest, beginning approximately, with the summer until the middle of autumn, when the flowering begins again.

The study carried out on the fruiting period of this species shows that the period of primary dispersion runs from the end of March to June, presenting a period of maximum dispersion in June. Coinciding with a rise in temperature and a drop in relative humidity. This raises the relation existing between the first dispersion of this species and the action of the colonies of *Messor barbarus*. In the fruiting period, two problems arise: one, not all the fruits formed give rise to mature seeds, and two seeds that reach the soil undergo two actions of the insects, one of dispersion and one of predation. Two insects responsible for seed predation have been detected: *Sitona regensteinensis* and *Bruchidius lividimanus*, both of which are seminivorous on genistaceae. The first has been cited on *Cytisus scoparius*, *Cytisus purgans*, *Cytisus laburnum*, *Ulex europaeus* and *Ulex nanus*. Predation does not occur directly on the seed, but the insect deposits the laying, when the fruit is immature and the larvae develop inside the fruit, feeding on the seed until reaching maturity. This same process, but produced by corculionids, parasitic hymenoptera and lepidopteran larvae, has been observed in Ulex parviflorus (Herrera, 1985).

1. 3 specimens of *Ulex parviflorus* have been selected.
2. To obtain data on seed dispersion, the fallen seeds were collected in a 3m x 3m cloth, with a 0.5 mm diameter web, located under each study specimen.
3. From the collected seeds, the distance from them to the trunk of the plant has been measured, and the orientation has also been measured: E, SE, S, SW, W, NW, N and NE.
4. To avoid the modification of the dispersion of the seeds by the ants has been used a repellent of these.
5. Weekly periodicity from the maximum fruiting period.

5.6. Estimating the population of pollinators

The pollinators observed in the plot correspond to two species: *Apis mellifera L.* or common bee, is in charge of collecting the pollen of *Ulex parviflorus* in the plot of experimentation. Some specimens of *Bombus terrestris L.* have also been observed. There is a relationship between the periods of maximum bloom and pollination by bees, but taking into account that climatic conditions also influence the temperature, wind speed and relative air humidity.

1. 4 specimens of *Ulex parviflorus* have been selected.
2. Analysis of pollinating plant study: *Apis mellifera* (honeybee).
3. The following counts were made: number of visitors per plant, number of flowers visited by bee and number of flowers available per plant.
4. Observation period 15 minutes.
5. Weekly periodicity, during the 8 weeks of maximum flowering. (Dafni, 1992)

Of the flowers present in plants, bees visit about 15% of these. The study carried out on the plot on the number of flowers of the plant, number of visitors per plant and number of flowers visited reflects the following results (Table 14):

TABLE 14
Relationship of the bees with the plot

Observations (Weeks)	$\overline{X}$ visits/plant	$\overline{X}$ flowers visited	$\overline{X}$ flowers available
1	3	24.8	1156
2	0.5 (*)	9.5	971
3	0.75 (*)	2.6	739.5
4	4.25	14.82	570.5

5	5.25	13.04	411
6	5	18.3	325.5
7	6.25	16.88	288.75
8	0 (*)	0	268.25
9	0(?)	0	213.25

(*) It was very windy
(?)Cloudy day
Observations have been made in consecutive weeks.

- Visiting rate index is equal to the total number of visits in the observation period/number of flowers available in that period (Zimmerman[a], M., 1980)

- Effectiveness of the visit rate per flower and per unit time (Andersson, S., 1988)

$$V = \left(A \times N \right) / C \, (12)$$

A = Number of visitors per plant per unit of time.
N = Number of flowers visited per unit of time.
C = Number of flowers per plant.

- Attraction index equals the number of visitors/flowers available/time unit (Pleasants, J.M., 1980)

- Pollination efficiency equals the number of flowers visited per unit time (Richards, K.W., 1987)

The indices studied show the following data (Table 15):

TABLE 15
Table of indices

Observations (Weeks)	Visiting fee	V	Attraction index	Efficiency of pollination
1	0.064	0.773	0.0025	19.86
2	0.004	0.009	0.0005	1.266
3	0.002	0.006	0.0010	0.400
4	0'110	1.877	0.0074	16.80
5	0.166	3.5	0.0127	18.26
6	0.281	5.622	0.0153	24.40
7	0.365	9.134	0.0216	28.13
8	0	0	0	0
9	0	0	0	0

6. VALIDATION

Two validation criteria have been established:

1) Graphically: using a comparative analysis of the graphical silhouettes of the series of observed and calculated values.

2) Analytically: using a variant of the classical convergence norm (Martínez & Requena, 1986), through the following expression: $CV = \left| \frac{\overline{X}^0 - \overline{X}^C}{\overline{X}_0} \right| .100 \leq \varepsilon$

   CV being the validation criterion, $\overline{X}^0$ is the arithmetic mean of the observed values of the variable X, $\overline{X}^C$ is the arithmetic mean of the calculated values of the variable X by means of the model, and ε is a predetermined arbitrary absolute value. If this rule is verified, the model is considered validated. It is the same validation criterion used in the ULEX model (Usó-Domènech et al., 1995).

We consider as output variables, that is, those that must be validated or checked, the state variables of each submodels. Five specimens of the plant have been chosen with different sizes and therefore with different biomass. Validation has been performed for a period of 10 weeks witha 15% sample.

6.1. Validation of the submodel BULEX
The output variables of the BULEX submodel are: BT, BFL, BFRUT y BLIT.

1) Total biomass BT. The validation criteria are as follows: CV (plant 1) = 2.98; CV (plant 2) = 6; CV (plant 3) = 2.12; CV (plant 4) = 0.24; CV (plant 5) = 7.81. The comparison graphs will be observed in the figures: 6, 7, 8, 9 and 10.

   Observed biomass: -0-0-

   Calculated biomass: -x-x-

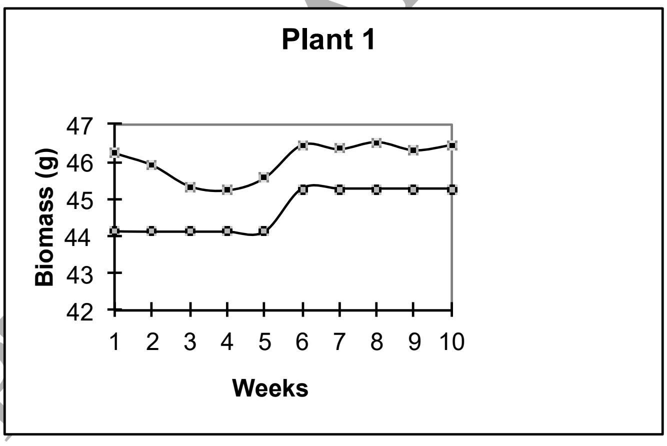

Figure 6. Plant 1: Validation Total Biomass

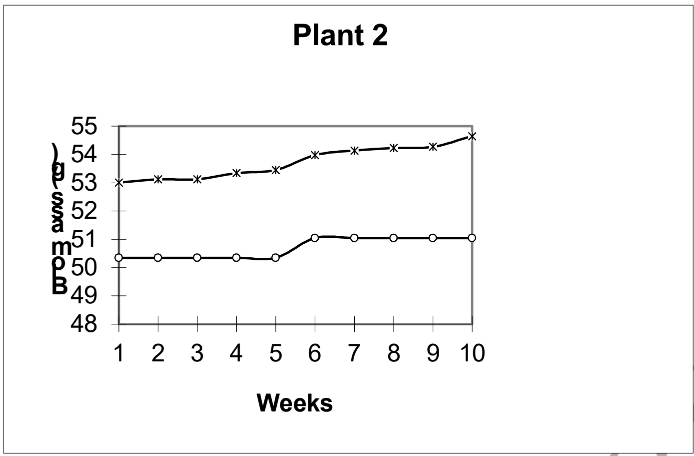

Figure 7. Plant 2: Validation Total Biomass

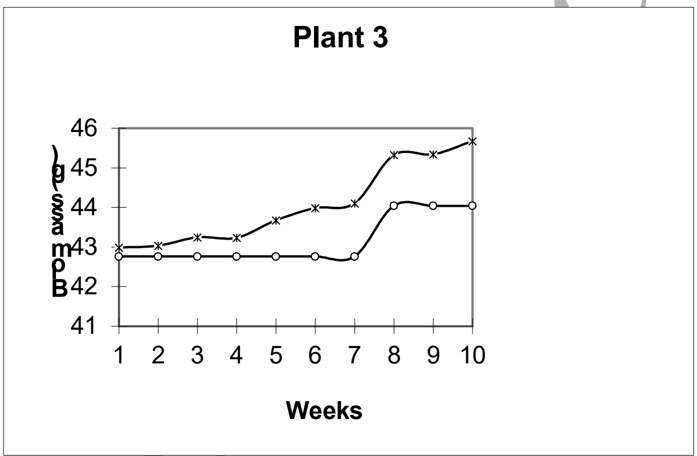

Figure 8. Plant 3: Validation Total Biomass

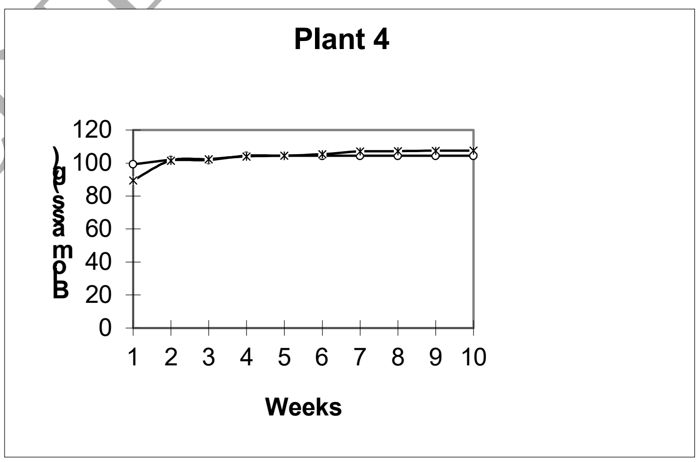

Figure 9. Plant 4: Validation Total Biomass

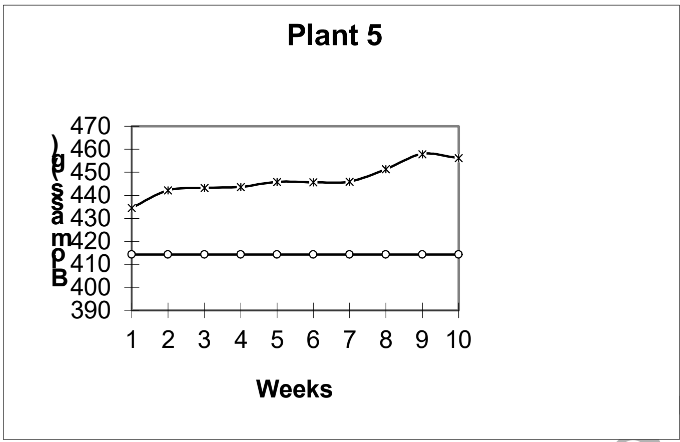

Figure 10. Plant 5: Validation Total Biomass

2) Floral Biomass BFL. The validation criteria are as follows: CV (plant 1) = 0.89; CV (plant 2) = 2.14; CV (plant 3) = 1.94; CV (plant 4) = 14.48; CV (plant 5) = 18.90. We will note that plant 5 is the only one that does not obey the established validation criteria.
The comparison graphs are given in the figures: 11, 12, 13, 14, 15.

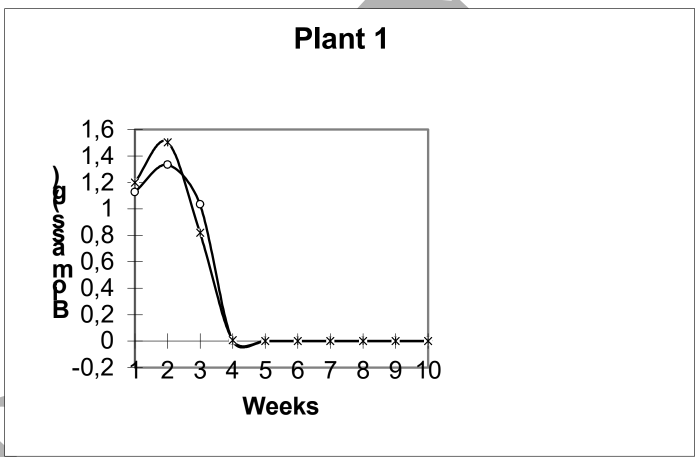

Figure 11. Plant 1: Validation Floral Biomass

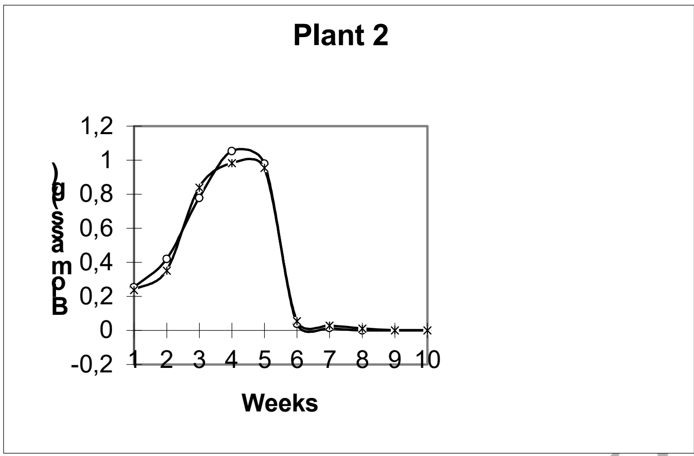

Figure 12. Plant 2: Validation Floral Biomass

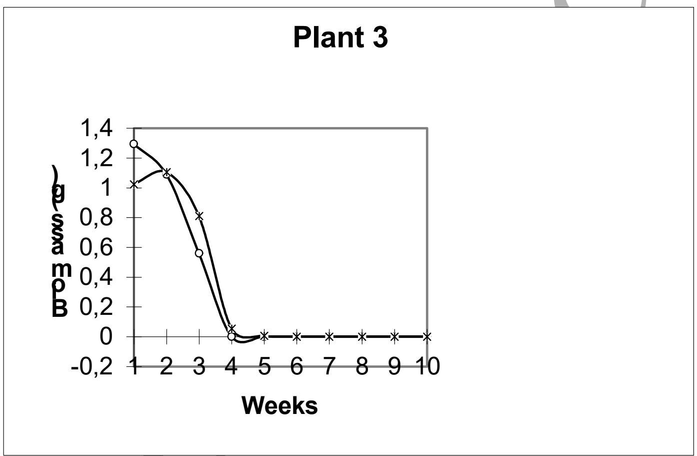

Figure 13. Plant 3: Validation Floral Biomass

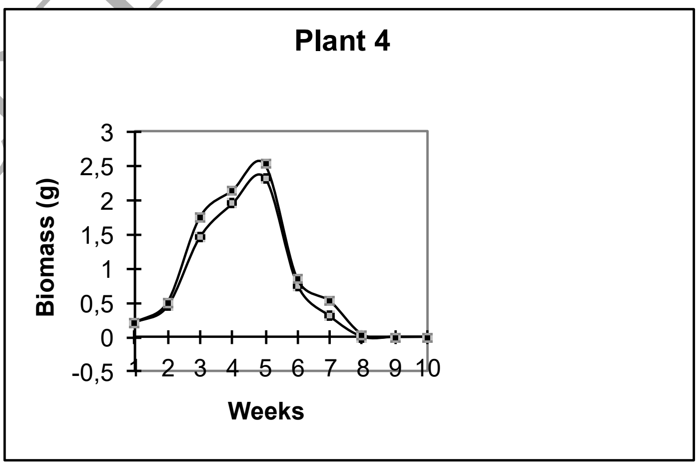

Figure 14. Plant 4: Validation Floral Biomass

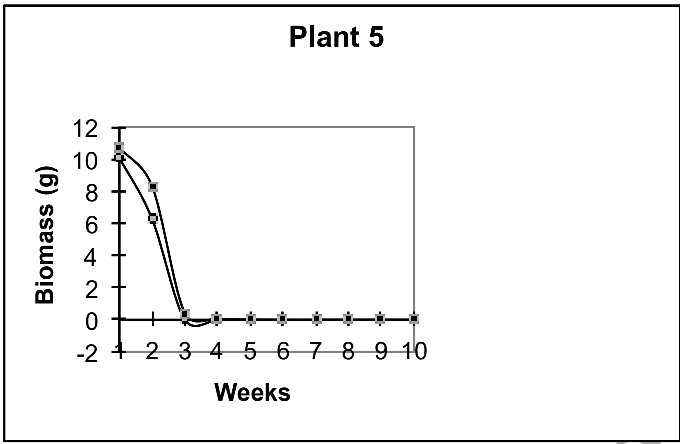

Figure 15. Plant 2: Validation Floral Biomass

3) Fruit biomass BFRUT. The validation criteria are as follows: CV (plant 1) = 14.45; CV (plant 2) = 2.91; CV (plant 3) = 0.49; CV (plant 4) = 0.86; CV (plant 5) = 1.49.
   The comparison graphs are given in the figures: 16, 17, 18, 19, 20.

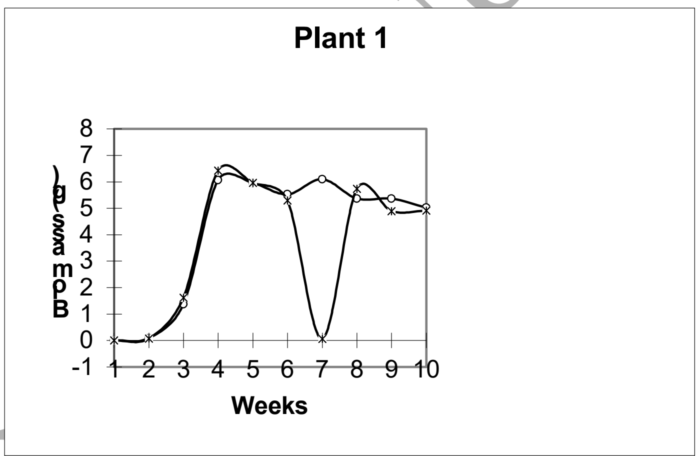

Figure 16. Plant 1: Validation Fruit Biomass

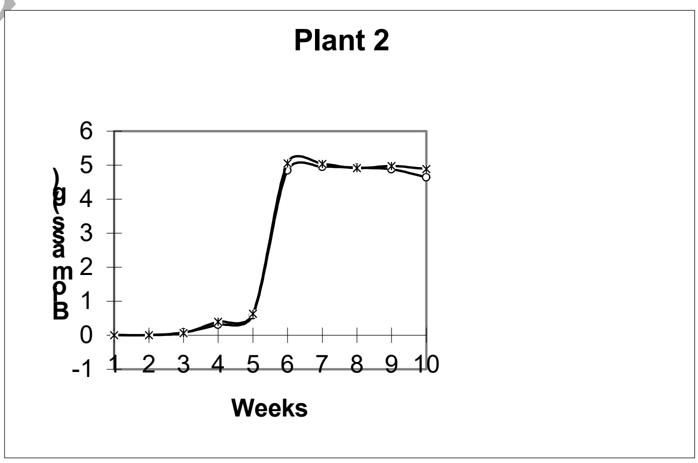

Figure 17. Plant 2: Validation Fruit Biomass

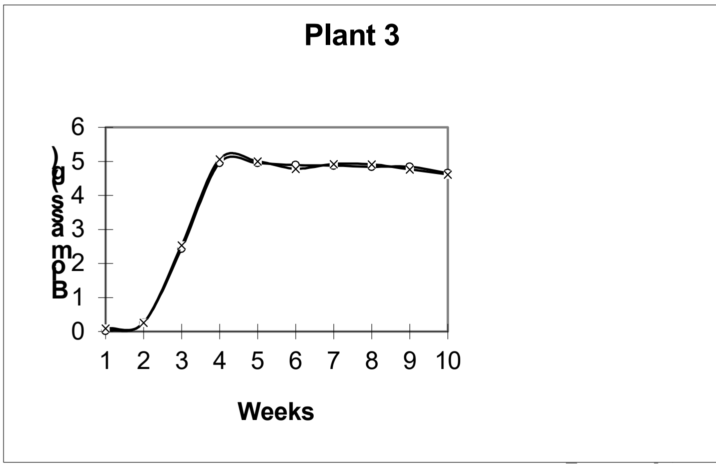

Figure 18. Plant 3: Validation Fruit Biomass

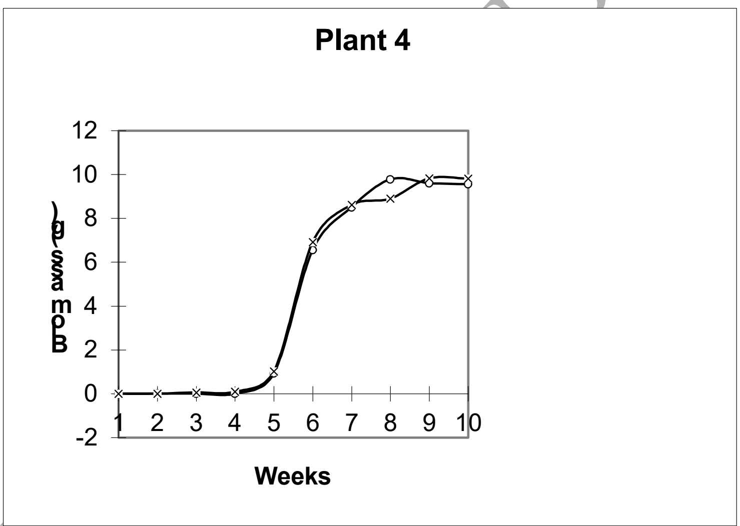

Figure 19 Plant 4: Validation Fruit Biomass

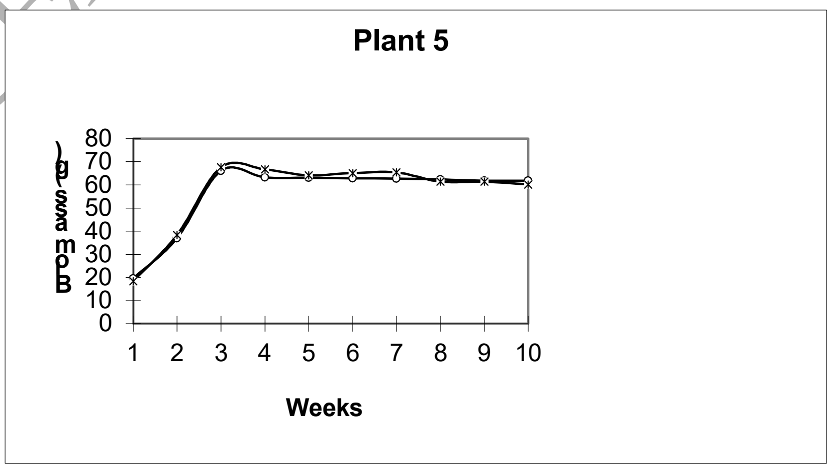

Figure 20. Plant 5: Validation Fruit Biomass

4) Biomass accumulated in the soil BLIT. The validation criteria are as follows: CV (plant 1) = 5.74; CV (plant 2) = 9.28; CV (plant 3) = 4.68; CV (plant 4) = 7.04: CV (plant 5) = 0.37.
   The comparison graphs are given in the figures: 21, 22, 23, 24, 25.

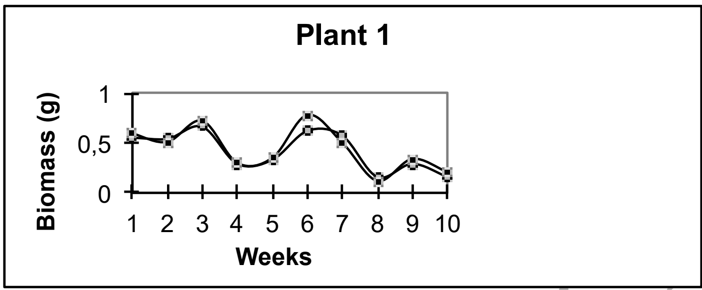

Figure 21. Plant 1: Validation Biomass accumulated in the soil

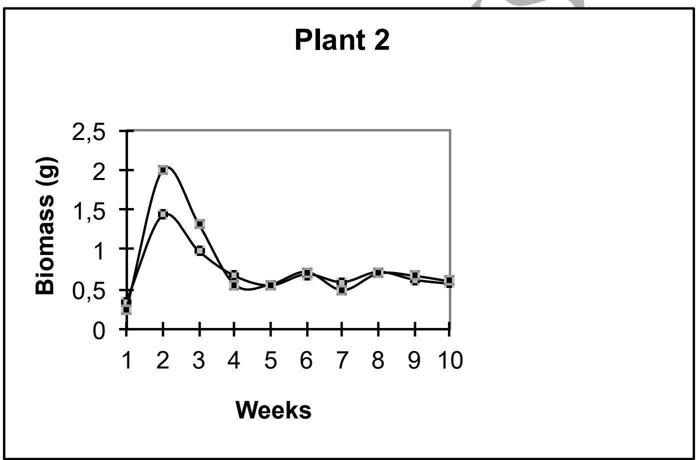

Figure 22. Plant 2: Validation Biomass accumulated in the soil

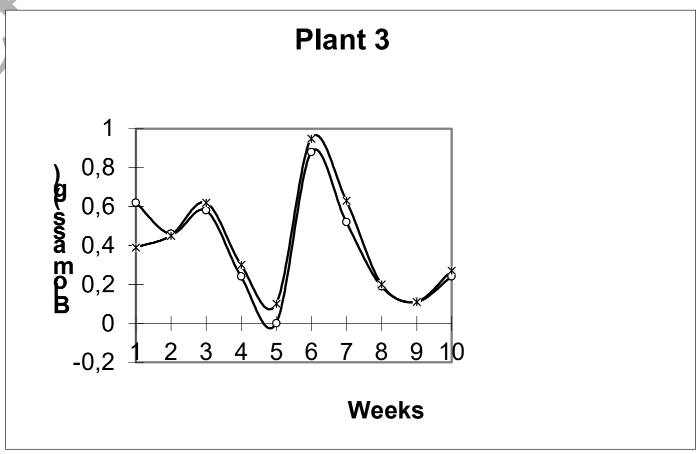

Figure 23. Plant 3: Validation Biomass accumulated in the soil

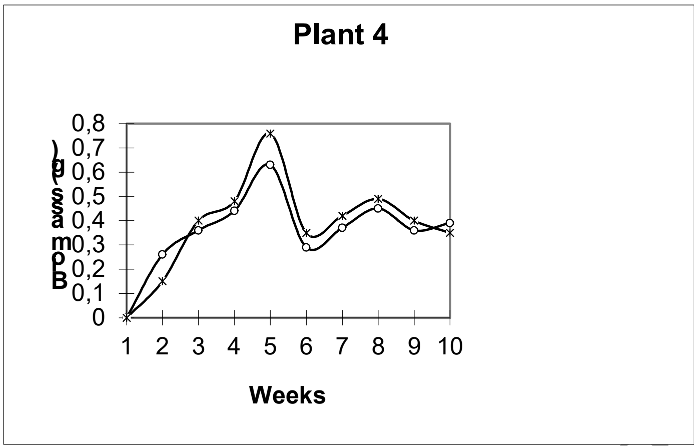

Figure 24. Plant 4: Validation Biomass accumulated in the soil

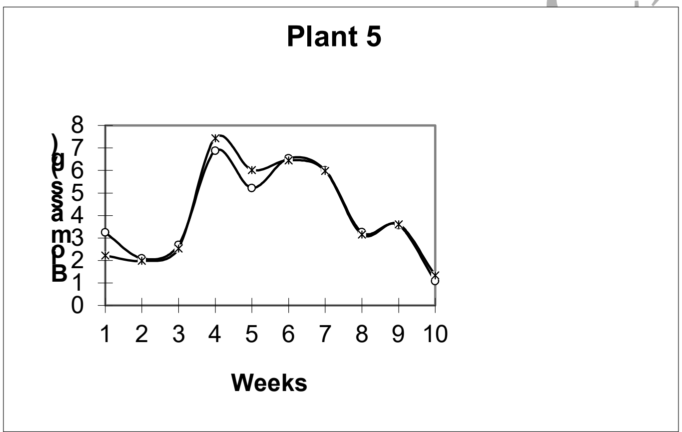

Figure 25. Plant 5: Validation Biomass accumulated in the soil

6.2. Validation of the submodel RPULEX

The output variables of the RPULEX submodel are: PGEM, PFLOR, and PFRUT. We will only validate these last two.

1) Population of flowers PFLOR. The validation criteria are as follows: CV (plant1) = 0.89; CV (plant 2) = 2.14; CV (plant 3) = 1.94; CV (plant 4) = 14.48; CV (plant 5) = 18.90.
   The comparison graphs are given in the figures: 26, 27, 28, 29, 30.

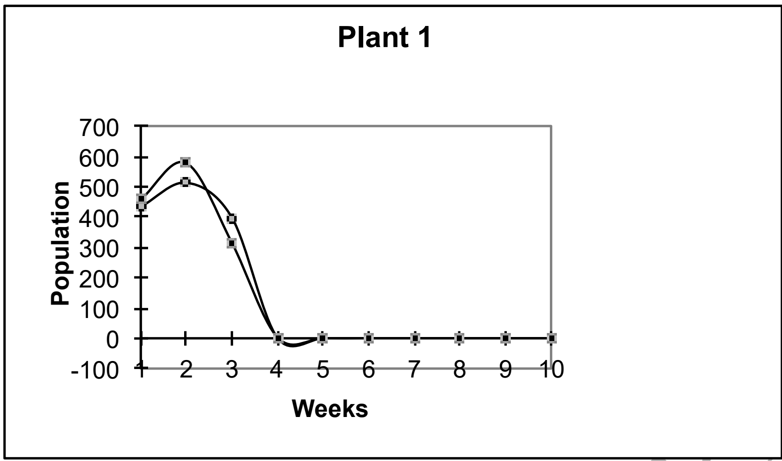

Figure 26. Plant 1: Validation population of flowers

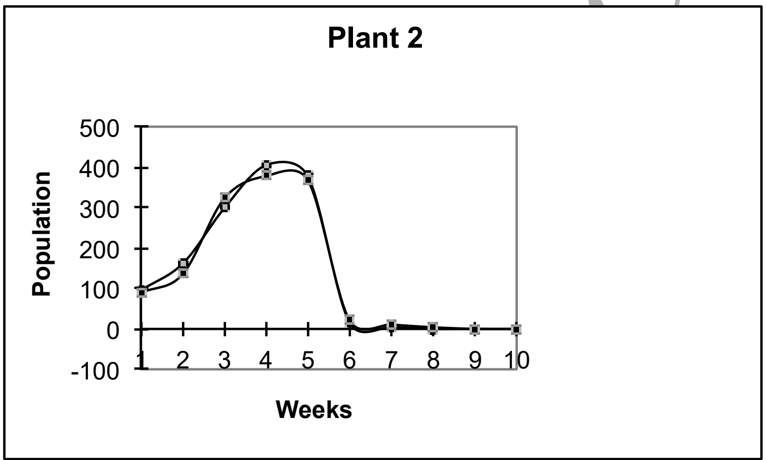

Figure 27. Plant 2: Validation population of flowers

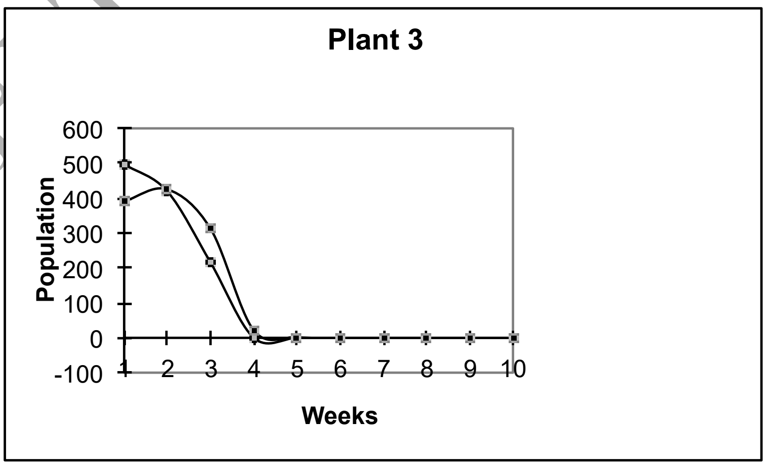

Figure 28. Plant 3: Validation population of flowers

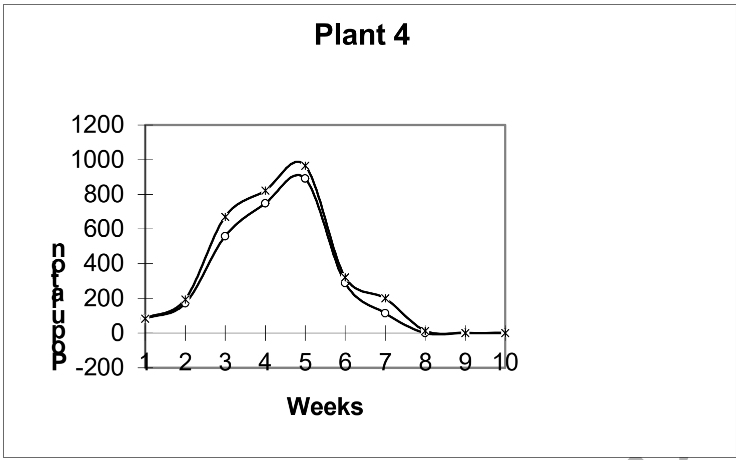

Figure 29. Plant 4: Validation population of flowers

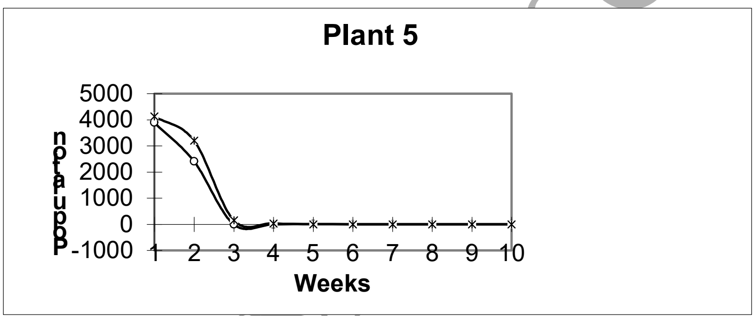

Figure 30. Plant 5: Validation population of flowers

2) Fruit population PFRUT. The validation criteria are as follows: CV (plant 1) = 14.8; CV (plant 2) = 2.47; CV (plant 3) = 0.07; CV (plant 4) = 3.13; CV (plant 5) = 1.57.
   The comparison graphs are given in the figures: 31, 32, 33, 34, 35.

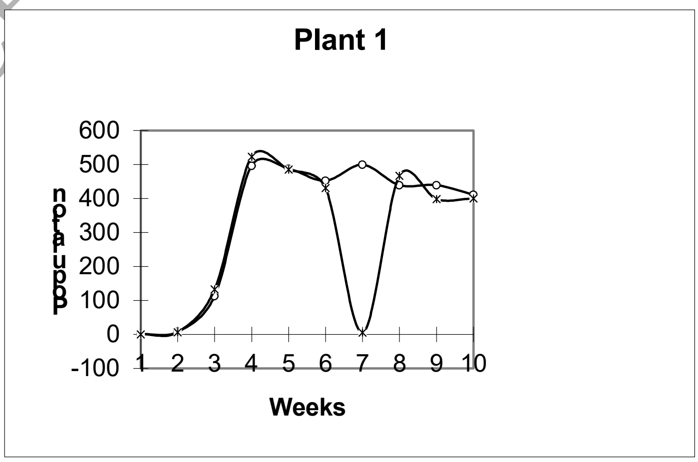

Figure 31. Plant 1: Validation fruit population

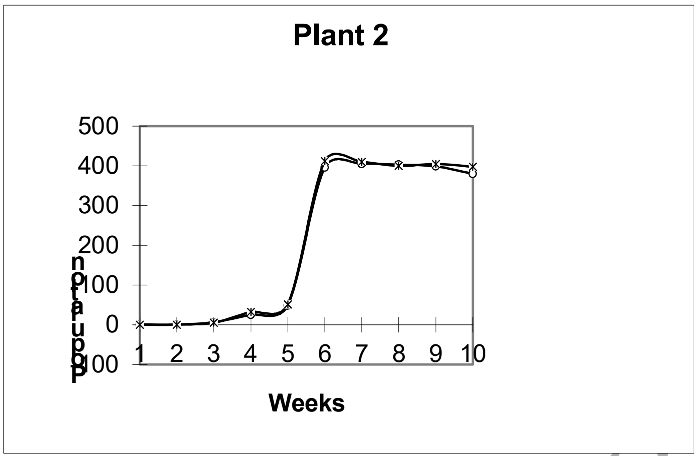

Figure 32. Plant 2: Validation fruit population

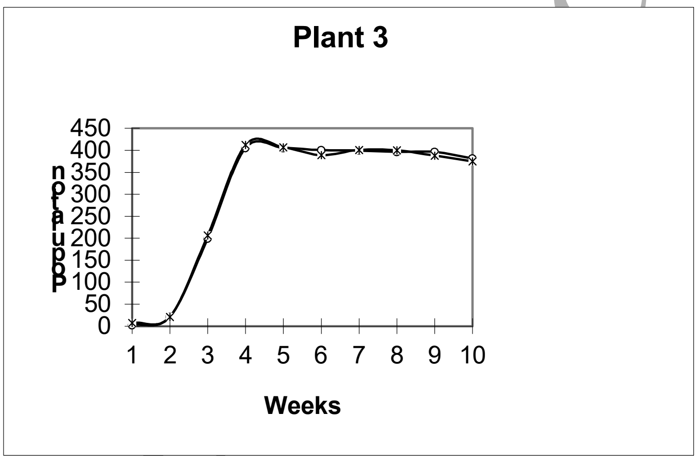

Figure 33. Plant 3: Validation fruit population

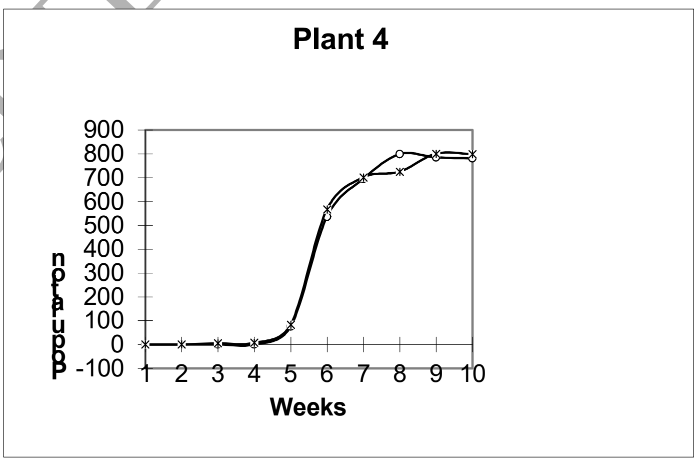

Figure 34. Plant 4: Validation fruit population

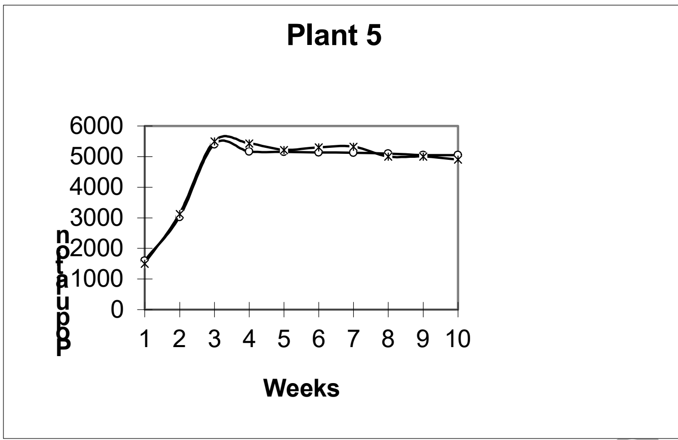

Figure 35. Plant 5: Validation fruit population

6.3. Validation of the population submodel
We could not validate this submodel because to do so we would have had to experiment for another year.

In short, we can say that the degree of adjustment is high. Therefore, we can consider the model as validated.

7. SIMULATION IN CONDITIONS OF DESERTIFICATION

In the Mediterranean, there are two natural factors that are the main causes of forest fires: climate and vegetation. The Mediterranean climate has shown strong seasonal contrasts and great variability over the years. Rainfall is usually concentrated in the spring and particularly in the autumn, often in a few highly erosive episodes. The rest of the year is dry. A long, hot and extremely dry summer turns the soil and vegetation into moisture-free material. These characteristics are favorable conditions for the occurrence of fires. As a result of these climatic processes vegetation presents certain characteristics, such as leaves, resins and essential oils, accumulation of dead biomass in the soil, etc. All this greatly increases the flammability of the territory. These climatic conditions can be made more extreme due to the presumed global climate change. An increase in average temperatures by a few degrees, a decrease in environmental humidity and a change in the direction of the winds will cause acceleration of the processes described above with an increase in forest fires. The loss of biomass results in a feedback process, a further increase in temperature, a decrease in humidity, etc., in an infernal cycle which logically ends up converting what were previously rich agricultural lands and Mediterranean mountain ecosystems into an appendix of the Sahara (Balairon, 1997; Barberá, López-Bermúdez, and Romero-Díaz, 1997; CICYT. 1997; López-Bermúdez, 1996). To establish the validity of this hypothesis, we will use a simulation in the ULEX model for a period of 24 weeks, with the following conditions:

1) Abnormally high atmospheric temperatures (an average increase of 2-3 ° C).
2) Ambient humidity abnormally low (A maximum of 60% and a minimum of 20%).
3) Pluviometry almost non-existent.
4) The wind speed does not change.
5) Wind directions change mostly at 180º and 270º.
6) Solar radiation is considered constant.

We will choose the following output variables: Total biomass (BT), flower population (PFLOR) and fruit population (PFRUT). These three variables have been chosen and not others, because they are considered as objective variables of the model, that is, influencing the behavior of the plant under conditions of climate change with a trend towards desertification. Although these conditions have been simulated with different plants of different sizes, we will include here three specimens that we consider significant. The population model has not been simulated. And it has been performed consciously, because the birth of the plant may depend on the population of ants, which are the main secondary dispersers. We do not know how these climatic conditions can affect this animal species. Therefore, to simulate what could happen in the population of *Ulex parviflorus* in conditions of desertification would be nothing more than mere lucubration. The examination of the extended model with a germination rate dependent on the ant population is another study.
It is difficult to pinpoint the reliability of predictions made on total biomass, and flower and fruit populations. However, you can also exercise the right to doubt. The pollination of the flowers depends mainly on the presence of bees. Nor can one know how this would affect the living conditions of this species, together with climate change and therefore pollination and fruit production. However, the results obtained seem logical and quite comparable to those obtained by the MARIOLA model (Usó-Domènech et al., 1995) simulating several Mediterranean shrub species with the same conditions established for the ULEX model.

7.1. Simulation of total biomass (BT)

The behavior of the total biomass is specified in the figures 36, 37 and 38.

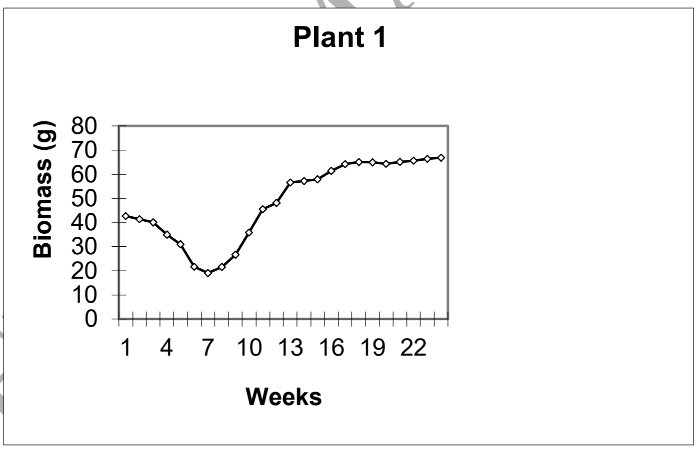

Figure 36. Plant 1: Simulation Total Biomass

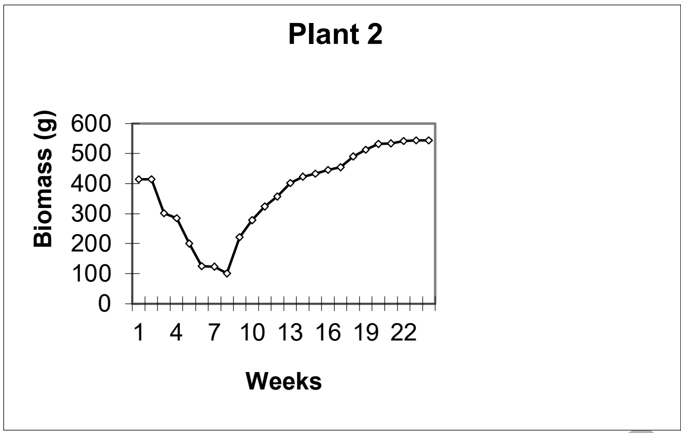

Figure 37. Plant 2: Simulation Total Biomass

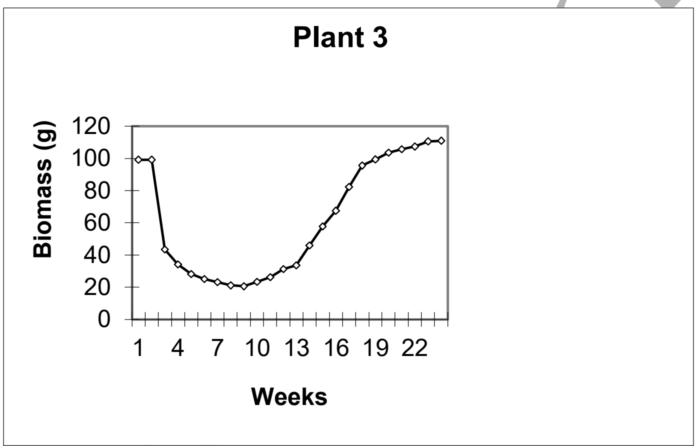

Figure 38. Plant 3: Simulation Total Biomass

We will observe a sharp decrease in biomass between month 1 and month 9. From this month, biomass begins to increase, reaching the initial level on month 13, and from there, it increases until it exceeds the initial biomass. On the third plant, the equalization occurs at month 17. The possible explanation for this fact is a loss of green biomass (spines) due to adverse conditions and an increase of the woody biomass (trunk and branches). The decrease of humidity is mainly due to the lack of rainfall and the wind coming from the South and West. A similar fact was observed in the simulation performed for *Cistus albidus* with the MARIOLA model (Usó-Domènech et al., 1995), with the difference that the smaller plants died there, which is not the case here. Possibly the explanations of the phenomenon are the condition of colonizing plants after a fire and its characteristic of being a pyrophite.

7.2. Simulation of the flower population (PFLOR)

The behavior of the flower population is specified in the figures 39, 40 and 41.

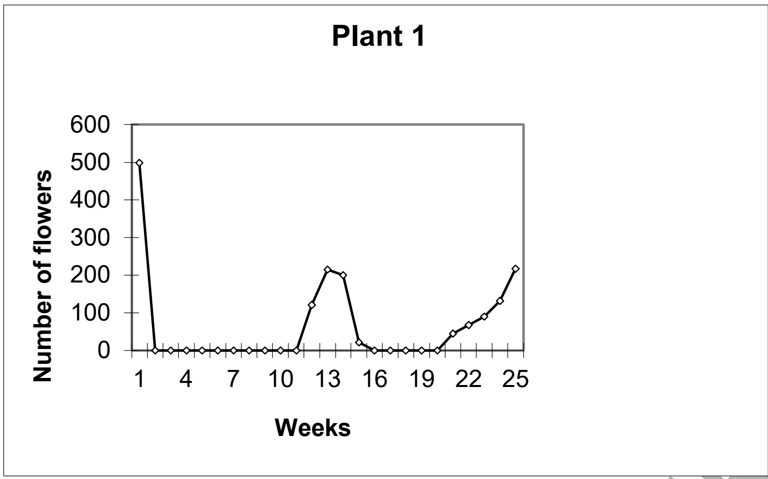

Figure 39. Plant 1: Simulation Flower Population

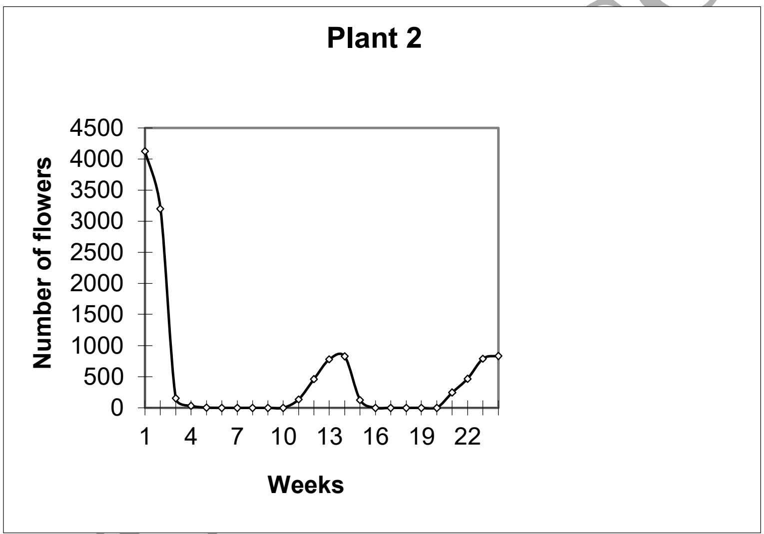

Figure 40. Plant 2: Simulation Flower Population

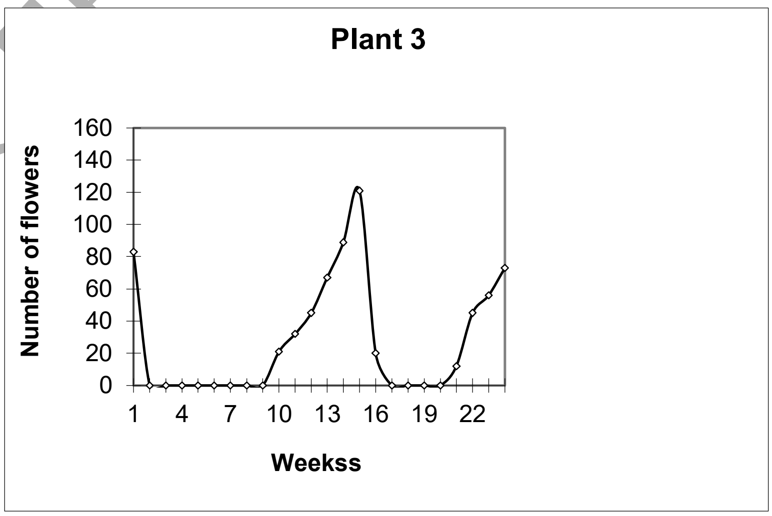

Figure 41. Plant 3: Simulation Flower Population

7.3. Simulation of the fruit population (PFRUT)
The behavior of the flower population is specified in the figures 42, 43 and 44.

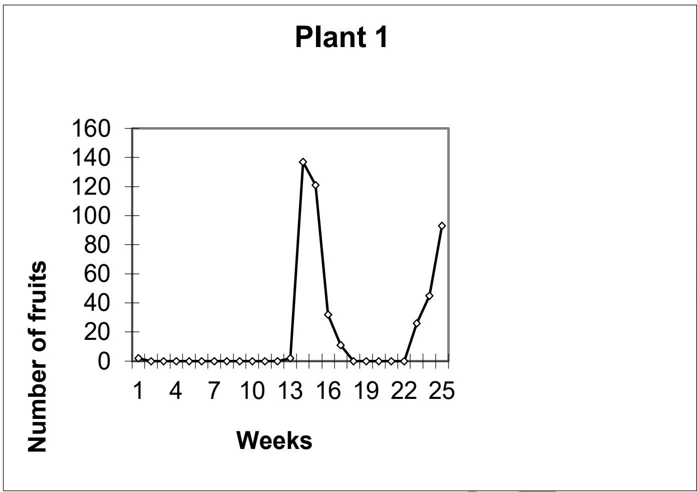

Figure 42. Plant 1: Simulation Fruit Population

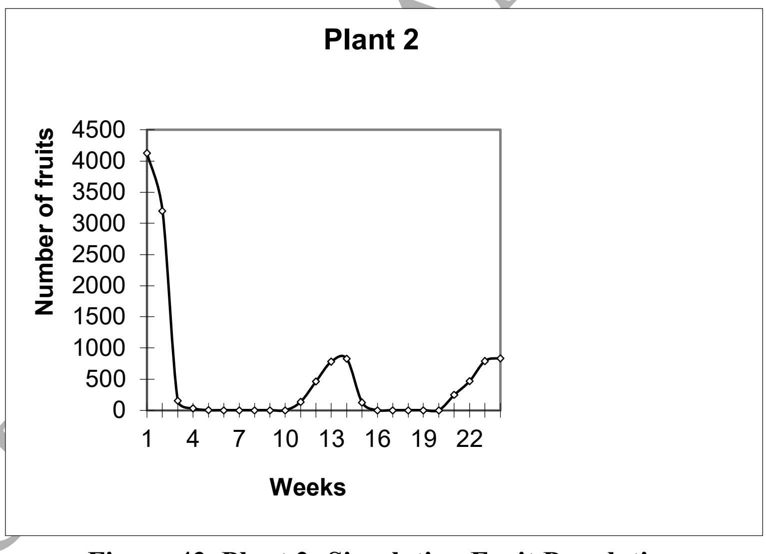

Figure 43. Plant 2: Simulation Fruit Population

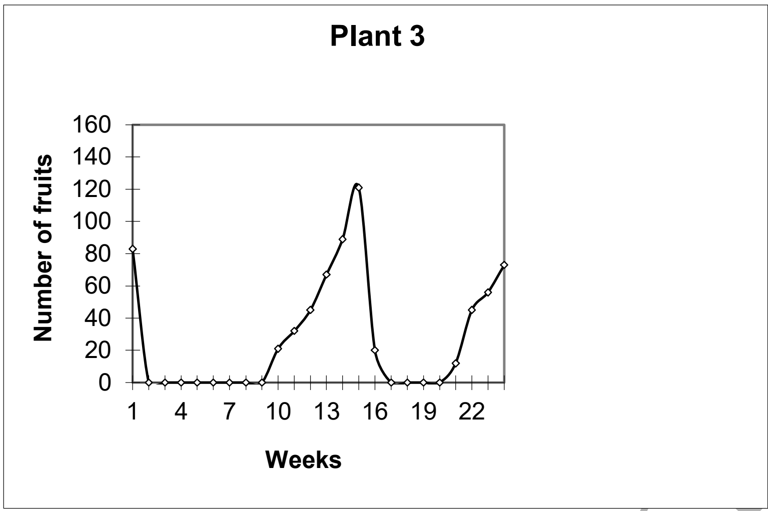

Figure 44. Plant 3: Simulation Fruit Population

We will observe, in the simulation of the floral population as in the fruit population, a sudden drop of reproductive elements in the second week. However, as of week 12, a recovery begins, peaking at weeks 16 to 18. Two successive periods of flowering and fruiting occur. However, the number of reproductive elements does not reach the existing level under normal conditions.

8. CONCLUSIONS

The ULEX model has allowed in these validated submodels a quite reliable calculation of a whole series of variables that determine the behavior of a Mediterranean pyrophite plant. The model allows one not only to follow behavioral evolution at the ecosystem level, but it can also be used to monitor the behavior of individual plants under natural conditions. However, the model is not complete. For it to be complete, the model should be extended to all significant species of the ecosystem under study. This requires three things: specialized staff, time and money. It must be admitted that none of the three things are available. However, a work plan can be outlined which would consist of the following steps:

1) Recognition and classification of all plant species that make up the ecosystem under study.
2) Recognition and classification of all animal species interacting with primary producers.
3) Obtaining the biomass equations of each of the plant species.
4) Field experimentation of the main variables that compose the model.
5) Sampling and soil analysis at different levels up to 50 cm in depth. In such chemical analyzes are the most important limiting factors, i.e., nitrogen, potassium, phosphorus, magnesium would be obtained and in those soils that did not have a calcareous substrate, calcium.
6) Measurement of other soil parameters such as pH, field capacity, ion exchange, pore volume, etc.
7) Estimation of the animal population by means of statistical methods.
8) Study of pests and diseases affecting plant species and calculation of biomass affected by the pest and/or disease.

Only by carrying out this ambitious plan of experimentation with a suitable human support team and with suitable material, could the ULEX model develop and demonstrate its full capabilities. However, in some submodels that we have experimented with, and calibrated and simulated, we believe we have satisfactorily demonstrated the usefulness of the ULEX model.

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ANEX A

1. SUBMODEL FOR REPRODUCTIVE BIOMASS BULEX

The state equations at the first level are:

$$\begin{cases} \dfrac{dBT}{dt} = CRBT - DBT \\ \dfrac{dBFL}{dt} = CRFL - DFL \\ \dfrac{dFRUT}{dt} = CRFRUT - DFRUT \\ \dfrac{dLIT}{dt} = CRLIT - DLIT \end{cases}$$

The flow equations are:

1) Growth equation of the total biomass

$$CRBT = -0.001575\,H.DIVI + 0.003424\,H.RS + 0.000612\,DIVI.RS - 171078760.103562\frac{1}{RS} + 31314.528762e^{-0.1RS}$$

$$-171096308.636654\,arctgRS + 2687573.27184662$$

Coefficient of determination $r^2 = 0.973137$
Multiple correlation coefficient $r = 0.986477$
Standard error $s = 0.478051$

2) Growth equation of floral biomass

$$CRFL = 0.157842\,H.T - 0.051445\,T.RS + 0.000331\,DIVI^2 + 710072137.579425\,\frac{1}{RS} - 137321.662847\,e^{-0.1RS}$$
$$+710138622.11609\,arctgRS - 1115482833.839448$$

Coefficient of determination r^2 = 0.869143
Multiple correlation coefficient r = 0.932278
Standard error s = 1.075721

3) Growth equation of the fruit biomass

$$CRFRUT = 0.151261\,H.T - 0.049267\,T.RS + 0.000317\,DIVI^2 + 679790796.889057\,\frac{1}{RS} - 131440.973861\,e^{-0.1RS}$$
$$+679854476.452949\,arctgRS - 1067912621.865067$$

Coefficient of determination r^2 = 0.827670
Multiple correlation coefficient r = 0.909764
Standard error s = 1.241713

4) Biomass growth fallen on the ground

$$CRLIT = 0.150288H.T - 0.048942T.RS + 0.000315DIVI^2 + 675277655{,}115520\frac{1}{RS} - 130567.956029e^{-0.1RS}$$
$$+ 675277655.115520arctgRS - 1060822736.275881$$

Coefficient of determination r^2 = 0.822248
Multiple correlation coefficient r = 0.906779
Standard error s = 1.260239

5) Decomposition of biomass fallen on the ground

$$DLIT = 0.008296DIVI + 0.007945RS + 260847589.139905\frac{1}{H} - 1968.712930\ln H +$$
$$261041052.330878arctgH - 410031447.305832$$

Coefficient of determination r^2 = 0.999925
Multiple correlation coefficient r = 0.999963
Standard error s = 0.025619

6) Destruction of total biomass

$$DBT = -0.158022T.RS + 1832545224.377605\frac{1}{RS} - 361940.900437e^{-0.1RS} + 7428.767143arctgT +$$
$$1832706412.761186arctgRS - 2878818413.808010$$

Coefficient of determination r^2 = 0.933082
Multiple correlation coefficient r = 0.965961
Standard error s = 6.718338

7) Destruction of floral biomass

$$DFL = -0.004611H^2 + 0.020724\,H.T - 0.001105\,RS.BT + 196614529.270695\,\frac{1}{BT} -$$
$$39359.95392786e^{-0.1BT} + 196626439.675833arctgBT - 308860028.912146$$

Coefficient of determination $r^2 = 0.845344$
Multiple correlation coefficient $r = 0.919426$
Standard error $s = 14.569866$

8) Destruction of fruit biomass

$$DFRUT = 0.000704H^2 - 0.013933H.T - 1.388284BT^2 + 64.859319BT -$$
$$774.905060e^{-0.1BT} - 9.463135\cos BT - 755.369054$$

Coefficient of determination $r^2 = 0.986841$
Multiple correlation coefficient $r = 0.993399$
Standard error $s = 2.244725$

The state equations at the second level are:

$$\begin{cases} \dfrac{dBT}{dt} = \begin{bmatrix} -0.001575H.DIVI + 0.003424H.RS + 0.000612DIVI.RS - 171078760.103562\dfrac{1}{RS} + 31314.528762e^{-0.1RS} \\ -171096308.636654arctgRS + 2687573.27184662 \end{bmatrix} - \\ \begin{bmatrix} -0.158022T.RS + 1832545224.377605\dfrac{1}{RS} - 361940.900437e^{-0.1RS} + 7428.767143arctgT + \\ 1832706412.761186arctgRS - 2878818413.808010 \end{bmatrix} \\ \dfrac{dBFL}{dt} = \begin{bmatrix} 0.157842H.T - 0.051445T.RS + 0.000331DIVI^2 + 710072137.579425\dfrac{1}{RS} - 137321.662847e^{-0.1RS} \\ + 710138622.11609arctgRS - 1115482833.839448 \end{bmatrix} - DFL \\ \dfrac{dFRUT}{dt} = \begin{bmatrix} 0.151261H.T - 0.049267T.RS + 0.000317DIVI^2 + 679790796.889057\dfrac{1}{RS} - 131440.973861e^{-0.1RS} \\ + 679854476.452949arctgRS - 1067912621.865067 \end{bmatrix} - DFRUT \\ \dfrac{dLIT}{dt} = \begin{bmatrix} 0.150288H.T - 0.048942T.RS + 0.000315DIVI^2 + 675277655{,}115520\dfrac{1}{RS} - 130567.956029e^{-0.1RS} \\ + 675277655.115520arctgRS - 1060822736.275881 \end{bmatrix} - \\ \begin{bmatrix} 0.008296DIVI + 0.007945RS + 260847589.139905\dfrac{1}{H} - 1968.712930\ln H + \\ 261041052.330878arctgH - 410031447.305832 \end{bmatrix} \end{cases}$$

The time unit is the week.

2. REPRODUCTIVE POPULATION SUBMODEL RPULEX

The state equations at the first level are:

$$\begin{cases} \dfrac{dPGEM}{dt} = NGEM - DGEM - NFLOR \\ \dfrac{dPFLOR}{dt} = NFLOR - DFLOR - NFRUT \\ \dfrac{dPFRUT}{dt} = NFRUT - DFRUT \end{cases}$$

The flow equations are:

1) Growth of flower buds

$$NGEM = 0.028815H^2 - 0.274987H.T + 0.004894H.DIVI + 0.065597H.VEVI - 3442.081547\frac{1}{T}$$
$$- 24.466093\ln DIVI - 3.188317\sqrt{BT} + 489.140451$$

Coefficient of determination r^2 = 0.514605
Multiple correlation coefficient r = 0.717360
Standard error s = 26.410942

2) Destruction of the flower buds
It could not be determined as there was no observed loss of flower buds.

3) Birth of flowers

$$NFLOR = 0.255629H^2 - 0.002933H.T + 0.000755DIVI.RS + 575641632.645585\frac{1}{H} - 0.035504e^{0.1H}$$
$$-575913004.496804\, arctgH + 904637494.901445$$

Coefficient of determination r^2 = 0.987305
Multiple correlation coefficient r = 0.993632
Standard error s = 0.188829

4) Destruction of flowers

$$DFLOR = -0.000019H^2 - 0.000144H.T + 0.000001H.DIVI - 0.004154H.VEVI + 0.000010PFLOR^2$$
$$- 2.287725e^{-0.1PFLOR} - 0.563307arctgPFLOR + 2.895160$$

Coefficient of determination r^2 = 0.651036
Multiple correlation coefficient r = 0.806868
Standard error s = 0.631142

5) Birth of fruits

$$NFRUT = 0.302882H^2 + 0.002635H.T - 0.000094DIVI.PFLOR - 660541528.335827\frac{1}{H}$$
$$- 0.041808e^{0.1H} - 660860389.628429\, arctgH + 1038071709.253941$$

Coefficient of determination r^2 = 0.671784
Multiple correlation coefficient r = 0.819624
Standard error s = 1.072221

6) Destruction of fruit

$$DFRUT = 0.046645H^2 - 0.705822H.T - 0.017749T.PFRUT + 0.000437PFRUT^2 - 11754.103208e^{-0.1H}$$
$$+ 361.791874\sqrt{T} - 879.398835$$

Coefficient of determination r^2 = 0.845344
Multiple correlation coefficient r = 0.919426
Standard error s = 25.305988

The state equations at the second level are:

$$\begin{cases} \dfrac{dPGEM}{dt} = \begin{bmatrix} 0.028815H^2 - 0.274987H.T + 0.004894H.DIVI + 0.065597H.VEVI - 3442.081547\dfrac{1}{T} \\ -24.466093\ln DIVI - 3.188317\sqrt{BT} + 489.140451 \end{bmatrix} - \\ DGEM - \begin{bmatrix} 0.255629H^2 - 0.002933H.T + 0.000755DIVI.RS + 575641632.645585\dfrac{1}{H} - 0.035504e^{0.1H} \\ -575913004.496804\,arctgH + 904637494.901445 \end{bmatrix} \\ \dfrac{dPFLOR}{dt} = \begin{bmatrix} 0.255629H^2 - 0.002933H.T + 0.000755DIVI.RS + 575641632.645585\dfrac{1}{H} - 0.035504e^{0.1H} \\ -575913004.496804\,arctgH + 904637494.901445 \end{bmatrix} - \\ \begin{bmatrix} -0.000019H^2 - 0.000144H.T + 0.000001H.DIVI - 0.004154H.VEVI + 0.000010PFLOR^2 \\ -2.287725e^{-0.1PFLOR} - 0.563307\,arctgPFLOR + 2.895160 \end{bmatrix} - \\ \begin{bmatrix} 0.302882H^2 + 0.002635H.T - 0.000094DIVI.PFLOR - 660541528.335827\dfrac{1}{H} \\ -0.041808e^{0.1H} - 660860389.628429\,arctgH + 1038071709.253941 \end{bmatrix} \\ \dfrac{dPFRUT}{dt} = \begin{bmatrix} 0.302882H^2 + 0.002635H.T - 0.000094DIVI.PFLOR - 660541528.335827\dfrac{1}{H} \\ -0.041808e^{0.1H} - 660860389.628429\,arctgH + 1038071709.253941 \end{bmatrix} - \\ \begin{bmatrix} 0.046645H^2 - 0.705822H.T - 0.017749T.PFRUT + 0.000437PFRUT^2 - 11754.103208e^{-0.1H} \\ +361.791874\sqrt{T} - 879.398835 \end{bmatrix} \end{cases}$$

The time unit is the week.

3. PLANT POPULATION SUBMODEL PULEX

The state equations at the first level are:

$$\begin{cases} \dfrac{dPSEM}{dt} = DISP - AGERM - GERM \\ \dfrac{dPGERM}{dt} = GERM - ABORT \\ \dfrac{dPOB}{dt} = NAC - MORT \end{cases}$$

The flow equations are:

1) Seed dispersion
$DISP = NSEM.PFRUT$

2) Non-germinating seeds
$AGERM = TAGERM.PSEM$

3) Seed germination
$GERM = TGERM.PSEM$

4) Abortion of germinated seeds

$$ABORT = 0.000529H^2 + 0.000509H.T + 0.000626DIVI.PLU - 0.006203PLU.PGERM + 424.924662e^{-0.1H} + 0.406379\cos T - 3.362289$$

Coefficient of determination $r^2 = 0.886138$
Multiple correlation coefficient $r = 0.941349$
Standard error $s = 0.406119$

5) Birth of plants

$$NAC = 0.000138H.T + 0.000453DIVI.PLU - 2.158918e^{0.1PGERM} + 0.621070\cos t + 3.073609$$

Coefficient of determination $r^2 = 0.737661$
Multiple correlation coefficient $r = 0.858872$
Standard error $s = 0.553228$

6) Death of plants

$$MORT = 0.002963H^2 - 0.000353T.DIVI + 0.000014DIVI^2 - O.265975VEVI^2 + 0.054229VEVI.POB - 0.451820H + 16.812900$$

Coefficient of determination $r^2 = 0.692884$
Multiple correlation coefficient $r = 0.832397$
Standard error $s = 0.324627$

The state equations in the second level are:

$$\begin{cases} \dfrac{dPSEM}{dt} = NSEM.PFRUT - TAGERM.PSEM - TGERM.PSEM \\ \dfrac{dPGERM}{dt} = TGERM.PSEM - \begin{bmatrix} 0.000529H^2 + 0.000509H.T + 0.000626DIVI.PLU \\ -0.006203PLU.PGERM \\ +424.924662e^{-0.1H} + 0.406379\cos T - 3.362289 \end{bmatrix} \\ \dfrac{dPOB}{dt} = \left[0.000138H.T + 0.000453DIVI.PLU - 2.158918e^{0.1PGERM} + 0.621070\cos t + 3.073609 \right] - \\ \begin{bmatrix} 0.002963H^2 - 0.000353T.DIVI + 0.000014DIVI^2 - O.265975VEVI^2 + 0.054229VEVI.POB - \\ 0.451820H + 16.812900 \end{bmatrix} \end{cases}$$

The time unit is the week.

NOTE
The authors are aware that functions like arctgX have difficult biological interpretation. However, the use of these functions increases the fit of the flow equation, and therefore, the reliability of the model. The problem was raised in other models that we did long ago, and we had to choose between the explanation and the adjustment. Not an easy decision. In the end we opted for the biggest adjustment. A long time ago we formulated a Uncertainty Principle (Usó-Domènech, Villacampa, Mateu, and Sastre-Vazquez, 2000.), where the problem was dealt with. We believe there may be an attempt to explain this fact.